Chapter Eight

Sections 8.1 – 8.3

8.1 An **estimator** is a sample statistic used to estimate a population parameter. The value(s) assigned to a population parameter based on the value of a sample statistic is called an **estimate**.

8.2 The value of a sample statistic used to estimate a population parameter is called a **point estimate**. Under this procedure, we assign a single value to the population parameter being estimated. In **interval estimation**, an interval is constructed around the point estimate, and it is stated that this interval is likely to contain the corresponding population parameter.

8.3 The sample mean $\bar{x}$ is the **point estimator of the population mean μ**. The **margin of error** is $E = z\sigma_{\bar{x}}$.

8.4 The **width of a confidence interval** may be decreased by lowering the confidence level or by increasing the sample size. Since lowering the confidence level results in a less reliable estimate, it is preferable to increase the sample size.

8.5 The width of a confidence interval depends on $E = z\sigma_{\bar{x}} = z\left(\sigma/\sqrt{n}\right)$. When n increases, $z\left(\sigma/\sqrt{n}\right)$ decreases and the width of the confidence interval decreases. From Example 8–1 in the text, $n = 25$, $\bar{x}$ = \$145, and σ = \$35. Then, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 35/\sqrt{25} = \$7$. The 90% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 145 \pm 1.65(7) = 145 \pm 11.55 = \$133.45 \text{ to } \$156.55$. The width of this interval is \$156.55 – \$133.45 = \$23.10. If $n = 100$, but all other values remain the same, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 35/\sqrt{100} = \$3.50$, and the 90% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 145 \pm 1.65(3.50) = 145 \pm 5.78 = \$139.22 \text{ to } \$150.78$. The width of this interval is \$150.78 – \$139.22 = \$11.56. Thus, the 90% confidence interval for μ is narrower when $n = 100$ than when $n = 25$.

8.6 The width of a confidence interval depends on $E = z\sigma_{\bar{x}} = z\left(\sigma/\sqrt{n}\right)$. When the confidence level decreases, z decreases. Hence, $z\left(\sigma/\sqrt{n}\right)$ decreases and the width of the confidence interval decreases. From Example 8–1 in the text, $n = 25$, $\bar{x}$ = \$145, and σ = \$35. Then $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 35/\sqrt{25} = \$7$, and the 95% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 145 \pm 1.96(7) = 145 \pm 13.72 = \$131.28$ to $\$158.72$. The width of this interval is \$158.72 – \$131.28 = \$27.44. If we decrease the confidence level to 90%, but all other values remain the same, the 90% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 145 \pm 1.65(7) = 145 \pm 11.55 = \$133.45$ to $\$156.55$.

The width of this interval is \$156.55 – \$133.45 = \$23.10. Thus, the 90% confidence interval for μ is narrower than the 95% confidence interval.

8.7 A **confidence interval** is an interval constructed around a point estimate. A **confidence level** indicates how confident we are that the confidence interval contains the population parameter.

8.8 The **margin of error for the estimate for μ** is the quantity that is subtracted from and added to the value of the sample statistic $\bar{x}$ to obtain a confidence interval for μ. It is given by $E = z\sigma_{\bar{x}}$, where z is determined by the confidence level.

8.9 If we take all possible samples of a given size and construct a 99% confidence interval for μ from each sample, we can expect about 99% of these confidence intervals will contain μ and 1% will not.

8.10
a. $z = 1.65$ b. $z = 1.96$ c. $z = 2.05$
d. $z = 2.17$ e. $z = 2.33$ f. $z = 2.58$

8.11 $n = 20$, $\bar{x} = 24.5$, $\sigma = 3.1$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 3.1/\sqrt{20} = .69318107$

a. $\bar{x} = 24.5$

b. The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 24.5 \pm 2.58(.69318107) = 24.5 \pm 1.79 = 22.71$ to 26.29

c. $E = z\sigma_{\bar{x}} = 2.58(.69318107) = 1.79$

8.12 $n = 81$, $\bar{x} = 48.25$, $\sigma = 4.8$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 4.8/\sqrt{81} = .53333333$

a. $\bar{x} = 48.25$

b. The 95% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 48.25 \pm 1.96(.53333333) = 48.25 \pm 1.05 = 47.20$ to 49.30

c. $E = z\sigma_{\bar{x}} = 1.96(.53333333) = 1.05$

8.13 $n = 36$, $\bar{x} = 74.8$, $\sigma = 15.3$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 15.3/\sqrt{36} = 2.55$

a. The 90% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 74.8 \pm 1.65(2.55) = 74.8 \pm 4.21 = 70.59$ to 79.01

b. The 95% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 74.8 \pm 1.96(2.55) = 74.8 \pm 5.00 = 69.80$ to 79.80

c. The 99% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 74.8 \pm 2.58(2.55) = 74.8 \pm 6.58 = 68.22$ to 81.38

d. Yes, the width of the confidence intervals increases as the confidence level increases. This occurs because as the confidence level increases, the value of z increases.

8.14 $n = 25$, $\bar{x} = 143.72$, $\sigma = 14.8$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 14.8/\sqrt{25} = 2.96$

a. The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 143.72 \pm 2.58(2.96) = 143.72 \pm 7.64 = 136.08 \text{ to } 151.36$

b. The 95% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 143.72 \pm 1.96(2.96) = 143.72 \pm 5.80 = 137.92 \text{ to } 149.52$

c. The 90% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 143.72 \pm 1.65(2.96) = 143.72 \pm 4.88 = 138.84 \text{ to } 148.60$

d. Yes, the width of the confidence intervals decreases as the confidence level decreases. This occurs because as the confidence level decreases, the value of z decreases.

8.15 $\bar{x} = 81.90$ and $\sigma = 6.30$

a. $n = 16$, so $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 6.30/\sqrt{16} = 1.575$

The 99% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 81.90 \pm 2.58(1.575) = 81.90 \pm 4.06 = 77.84 \text{ to } 85.96$

b. $n = 20$, so $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 6.30/\sqrt{20} = 1.40872283$

The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 81.90 \pm 2.58(1.40872283) = 81.90 \pm 3.63 = 78.27 \text{ to } 85.53$

c. $n = 25$, so $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 6.30/\sqrt{25} = 1.26$

The 99% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 81.90 \pm 2.58(1.26) = 81.90 \pm 3.25 = 78.65 \text{ to } 85.15$

d. Yes, the width of the confidence intervals decreases as the sample size increases. This occurs because the standard deviation of the sample mean decreases as the sample size increases.

8.16 $\bar{x} = 48.52$ and $\sigma = 7.14$

a. $n = 196$, so $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 7.14/\sqrt{196} = .51$

The 95% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 48.52 \pm 1.96(.51) = 48.52 \pm 1.00 = 47.52 \text{ to } 49.52$

b. $n = 100$, so $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 7.14/\sqrt{100} = .714$

The 95% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 48.52 \pm 1.96(.714) = 48.52 \pm 1.40 = 47.12 \text{ to } 49.92$

c. $n = 49$, so $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 7.14/\sqrt{49} = 1.02$

The 95% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 48.52 \pm 1.96(1.02) = 48.52 \pm 2.00 = 46.52 \text{ to } 50.52$

d. Yes, the width of the confidence intervals increases as the sample size decreases. This occurs because the standard deviation of the sample mean increases as the sample size decreases.

8.17 $n = 35$, $\bar{x} = (\sum x)/n = 1342/35 = 38.34$, $\sigma = 2.65$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.65/\sqrt{35} = .44793176$

a. $\bar{x} = 38.34$

b. The 98% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 38.34 \pm 2.33(.44793176) = 38.34 \pm 1.04 = 37.30$ to 39.38

c. $E = z\sigma_{\bar{x}} = 2.33(.44793176) = 1.04$

8.18 $n = 32$, $\bar{x} = (\sum x)/n = 2543/32 = 79.47$, $\sigma = 4.96$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 4.96/\sqrt{32} = .87681241$

a. $\bar{x} = 79.47$

b. The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 79.47 \pm 2.58(.87681241) = 79.47 \pm 2.26 = 77.21$ to 81.73

c. $E = z\sigma_{\bar{x}} = 2.58(.87681241) = 2.26$

8.19 a. $E = 2.50$, $\sigma = 12.5$, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.58)^2(12.5)^2}{(2.50)^2} = 166.41 \approx 167$$

b. $E = 3.20$, $\sigma = 12.5$, and $z = 2.05$ for 96% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.05)^2(12.5)^2}{(3.20)^2} = 64.13 \approx 65$$

8.20 a. $E = 5.50$, $\sigma = 14.50$, and $z = 2.33$ for 98% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.33)^2(14.50)^2}{(5.50)^2} = 37.73 \approx 38$$

b. $E = 4.25$, $\sigma = 14.50$, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.96)^2(14.50)^2}{(4.25)^2} = 44.72 \approx 45$$

8.21 a. $E = 2.3$, $\sigma = 15.40$, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.58)^2(15.40)^2}{(2.3)^2} = 298.42 \approx 299$$

b. $E = 4.1$, $\sigma = 23.45$, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.96)^2(23.45)^2}{(4.1)^2} = 125.67 \approx 126$$

c. $E = 25.9$, $\sigma = 122.25$, and $z = 1.65$ for 90% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.65)^2(122.25)^2}{(25.9)^2} = 60.65 \approx 61$$

8.22 a. $E = .17$, $\sigma = .90$, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.58)^2(.90)^2}{(.17)^2} = 186.56 \approx 187$$

b. $E = 1.45$, $\sigma = 5.82$, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.96)^2(5.82)^2}{(1.45)^2} = 61.89 \approx 62$$

c. $E = 5.65$, $\sigma = 18.20$, and $z = 1.65$ for 90% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.65)^2(18.20)^2}{(5.65)^2} = 28.25 \approx 29$$

8.23 $n = 1500$, $\bar{x} = \$299{,}720$, $\sigma = \$68{,}650$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 68{,}650/\sqrt{1500} = \$1772.535378$

The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 299{,}720 \pm 2.58(1772.535378) = 299{,}720 \pm 4573.14 = \$295{,}146.86$ to $\$304{,}293.14$

8.24 $n = 55$, $\bar{x} = 78.52$ pounds, $\sigma = 2.63$ pounds, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.63/\sqrt{55} = .35462913$ pounds

The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 78.52 \pm 2.58(.35462913) = 78.52 \pm .91 = 77.61$ to 79.43 pounds

8.25 $n = 14$, $\bar{x} = 53{,}550$ labor-hours, $\sigma = 7462$ labor-hours, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 7462/\sqrt{14} = 1994.303387$ labor-hours

a. The 98% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 53{,}550 \pm 2.33(1994.303387) = 53{,}550 \pm 4646.73 = 48{,}903.27$ to $58{,}196.73$ labor-hours

b. The sample mean of 53,550 labor-hours is an estimate of μ based on a random sample. Because of sampling error, this estimate might differ from the true mean, μ, so we make an interval estimate to allow for this uncertainty and sampling error.

8.26 $n = 20$, $\bar{x} = 36.02$ inches, $\sigma = .10$ inch, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .10/\sqrt{20} = .02236068$ inch

The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 36.02 \pm 2.58(.02236068) = 36.02 \pm .06 = 35.96$ to 36.08 inches

Since the upper limit, 36.08, is greater than 36.05, the machine needs an adjustment.

8.27 $n = 25$, $\bar{x} = 31.94$ ounces, $\sigma = .15$ ounce, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .15/\sqrt{25} = .03$ ounce

The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 31.94 \pm 2.58(.03) = 31.94 \pm .08 = 31.86$ to 32.02 ounces

Since the upper limit, 32.02, is less than 32.15, and the lower limit, 31.86, is greater than 31.85, the machine does not need an adjustment.

8.28 a. $n = 70$, $\bar{x} = \$420$, $\sigma = \$110$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 110/\sqrt{70} = \$13.1475147$

The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 420 \pm 2.58(13.1475147) = 420 \pm 33.92 = \$386.08 \text{ to } \$453.92$

b. The width of the confidence interval obtained in part a may be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate for μ.

8.29 a. $n = 120$, $\bar{x} = \$1575$, $\sigma = \$215$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 215/\sqrt{120} = \$19.62672498$

The 97% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 1575 \pm 2.17(19.62672498) = 1575 \pm 42.59 = \$1532.41 \text{ to } \$1617.59$

b. The width of the confidence interval obtained in part a may be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate for μ.

8.30 $E = \$2$, $\sigma = \$11$, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.96)^2(11)^2}{(2)^2} = 116.21 \approx 117$$

8.31 $E = .04$ ounce, $\sigma = .20$ ounce, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.58)^2(.20)^2}{(.04)^2} = 166.41 \approx 167$$

8.32 $E = \$3$, $\sigma = \$31$, and $z = 1.65$ for 90% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.65)^2(31)^2}{(3)^2} = 290.70 \approx 291$$

8.33 $E = .75$ hour, $\sigma = 2.5$ hours, and $z = 2.33$ for 98% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.33)^2(2.5)^2}{(.75)^2} = 60.32 \approx 61$$

8.34 To estimate μ, the mean commuting time at a 99% confidence level:

1. Take a random sample of 30 or more students from your school.
2. Determine the commuting time for each student.
3. Find $\bar{x}$, the mean of the sample.
4. Calculate $\sigma_{\bar{x}} = \sigma/\sqrt{n}$.
5. Calculate the 99% confidence interval for μ using the formula $\bar{x} \pm 2.58\sigma_{\bar{x}}$.

8.35 To estimate μ, the mean age of cars at a 95% confidence level:

1. Take a random sample of 30 or more U.S. car owners.
2. Determine the age of each car.
3. Find $\bar{x}$, the mean of the sample.
4. Calculate $\sigma_{\bar{x}} = \sigma/\sqrt{n}$.
5. Calculate the 95% confidence interval for μ using the formula $\bar{x} \pm 1.96\sigma_{\bar{x}}$.

Section 8.4

8.36 The following are the **similarities** between the standard normal distribution and the *t* distribution:

1. Both distributions are symmetric (bell–shaped) about the mean.
2. Neither distribution meets the horizontal axis.
3. Total area under each of these curves is 1.0.
4. The mean of both of these distributions is zero.

The following are the main **differences** between the standard normal distribution and the *t* distribution:

1. The *t* distribution has a lower height and a wider spread than the normal distribution.
2. The standard deviation of the standard normal distribution is 1 and that of the *t* distribution is $\sqrt{df/(df-2)}$, which is always greater than 1.
3. The *t* distribution has only one parameter, the degrees of freedom, whereas the (standard) normal distribution has two parameters, μ and σ.

8.37 The normal distribution has two **parameters**: μ and σ. Given the values of these parameters for a normal distribution, we can find the area under the normal curve between any two points. The *t* distribution has only one **parameter**: the degrees of freedom. The shape of the *t* distribution curve is determined by the degrees of freedom.

8.38 The number of **degrees of freedom** is defined as the number of observations that can be chosen freely. As an example, suppose the mean score of five students on an examination is 81. Consequently, the sum of these five scores is 81(5) = 405. Now, how many scores, out of five, can we choose freely so that the sum of these five scores is 405? The answer is that we are free to choose 5 – 1 = 4 scores.

Indeed, suppose we choose 87, 73, 69, and 94 as the four scores. Given these four scores and the information that the mean of the five scores is 81, the fifth score is: $405 - 87 - 73 - 69 - 94 = 82$. Thus, once we have chosen four scores, the fifth score is automatically determined. Hence, the number of degrees of freedom for this example is $df = n - 1 = 5 - 1 = 4$. We subtract 1 from n to obtain the degrees of freedom because we lose one degree of freedom to calculate the mean.

8.39 The t distribution is used to construct a confidence interval for the population mean μ if the following assumptions hold true:

Case I:

1. The population standard deviation σ is not known.
2. The sample size is small (i.e., $n < 30$).
3. The population from which the sample is drawn is (approximately) normally distributed.

Case II:

1. The population standard deviation σ is not known.
2. The sample size is large (i.e., $n \geq 30$).

8.40
a. $t = 1.782$
b. $df = n - 1 = 66 - 1 = 65$ and $t = -1.997$
c. $t = -3.265$
d. $df = n - 1 = 24 - 1 = 23$ and $t = 2.807$

8.41
a. $df = n - 1 = 21 - 1 = 20$ and $t = -1.325$
b. $df = n - 1 = 14 - 1 = 13$ and $t = 2.160$
c. $t = 3.281$
d. $t = -2.715$

8.42
a. Area in the right tail = .01
b. Area in the left tail = .05
c. $df = n - 1 = 55 - 1 = 54$, area in the left tail = .005
d. $df = n - 1 = 23 - 1 = 22$, area in the right tail = .01

8.43
a. Area in the left tail = .10
b. $df = n - 1 = 25 - 1 = 24$, area in the right tail = .005
c. $df = n - 1 = 9 - 1 = 8$, area in the right tail = .10
d. Area in the left tail = .01

8.44
a. $\alpha/2 = .5 - (.99/2) = .005$ and $t = 3.012$
b. $df = n - 1 = 36 - 1 = 35$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.030$
c. $\alpha/2 = .5 - (.90/2) = .05$ and $t = 1.746$

8.45
a. $df = n - 1 = 22 - 1 = 21$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.080$
b. $\alpha/2 = .5 - (.90/2) = .05$ and $t = 1.671$
c. $df = n - 1 = 24 - 1 = 23$, $\alpha/2 = .5 - (.99/2) = .005$, and $t = 2.807$

8.46 $n = 18$, $\sum x = 480.7$, and $\sum x^2 = 13{,}045.11$

$\bar{x} = (\sum x)/n = 480.7/18 = 26.71$

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{13{,}045.11 - \frac{(480.7)^2}{18}}{18-1}} = 3.49579112$$

$s_{\bar{x}} = s\big/\sqrt{n} = 3.49579112\big/\sqrt{18} = .82396587$

a. $\bar{x} = 26.71$

b. $df = n - 1 = 18 - 1 = 17$, $\alpha/2 = .5 - (.99/2) = .005$, and $t = 2.898$

The 99% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 26.71 \pm 2.898(.82396587) = 26.71 \pm 2.39 = 24.32$ to 29.10

c. $E = ts_{\bar{x}} = 2.898(.82396587) = 2.39$

8.47 $n = 11$, $\sum x = 15.5$, and $\sum x^2 = 534.49$

$\bar{x} = (\sum x)/n = 15.5/11 = 1.41$

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{534.49 - \frac{(15.5)^2}{11}}{11-1}} = 7.15995175$$

$s_{\bar{x}} = s\big/\sqrt{n} = 7.15995175\big/\sqrt{11} = 2.15880668$

a. $\bar{x} = 1.41$

b. $df = n - 1 = 11 - 1 = 10$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.228$

The 95% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 1.41 \pm 2.228(2.15880668) = 1.41 \pm 4.81 = -3.40$ to 6.22

c. $E = ts_{\bar{x}} = 2.228(2.15880668) = 4.81$

8.48 $n = 16$, $\bar{x} = 68.50$, $s = 8.9$, $s_{\bar{x}} = s\big/\sqrt{n} = 8.9\big/\sqrt{16} = 2.225$, and $df = n - 1 = 16 - 1 = 15$

a. $\alpha/2 = .5 - (.95/2) = .025$ and $t = 2.131$

The 95% confidence interval for μ is $\bar{x} \pm ts_{\bar{x}} = 68.50 \pm 2.131(2.225) = 68.50 \pm 4.74 = 63.76$ to 73.24

b. $\alpha/2 = .5 - (.90/2) = .05$ and $t = 1.753$

The 90% confidence interval for μ is $\bar{x} \pm ts_{\bar{x}} = 68.50 \pm 1.753(2.225) = 68.50 \pm 3.90 = 64.60$ to 72.40

The width of the 90% confidence interval for μ is smaller than that of the 95% confidence interval. This is so because the value of t for a 90% confidence level is smaller than that for a 95% confidence level (with df remaining the same).

c. $s_{\bar{x}} = s\big/\sqrt{n} = 8.9\big/\sqrt{25} = 1.78$

$df = n - 1 = 25 - 1 = 24$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.064$

The 95% confidence interval for μ is $\bar{x} \pm t s_{\bar{x}} = 68.50 \pm 2.064(1.78) = 68.50 \pm 3.67 = 64.83$ to 72.17

The width of the 95% confidence interval for μ is smaller with $n = 25$ than that of the 95% confidence interval with $n = 16$. This is so because the value of the standard deviation of the sample mean decreases as the sample size increases.

8.49 $n = 47$, $\bar{x} = 25.5$, $s = 4.9$, $s_{\bar{x}} = s/\sqrt{n} = 4.9/\sqrt{47} = .71473846$, and $df = n - 1 = 47 - 1 = 46$

a. $\alpha/2 = .5 - (.95/2) = .025$ and $t = 2.013$

The 95% confidence interval for μ is

$\bar{x} \pm t s_{\bar{x}} = 25.5 \pm 2.013(.71473846) = 25.5 \pm 1.44 = 24.06$ to 26.94

b. $\alpha/2 = .5 - (.99/2) = .005$ and $t = 2.687$

The 99% confidence interval for μ is

$\bar{x} \pm t s_{\bar{x}} = 25.5 \pm 2.687(.71473846) = 25.5 \pm 1.92 = 23.58$ to 27.42

The width of the 99% confidence interval for μ is larger than that of the 95% confidence interval. This is so because the value of t for a 99% confidence level is larger than that for a 95% confidence level.

c. $s_{\bar{x}} = s/\sqrt{n} = 4.9/\sqrt{32} = .86620581$

$df = n - 1 = 32 - 1 = 31$, $\alpha/2 = .5 - (.95/2) = .025$ and $t = 2.040$

The 95% confidence interval for μ is

$\bar{x} \pm t s_{\bar{x}} = 25.5 \pm 2.040(.86620581) = 25.5 \pm 1.77 = 23.73$ to 27.27

The width of the 95% confidence interval for μ is larger with $n = 32$ than that of the 95% confidence interval with $n = 47$. This is so because the value of the standard deviation of the sample mean increases as sample size decreases.

8.50 In each of the following parts, since $n = 100$ is very large, we use the normal distribution to approximate the t distribution, and construct the confidence interval using $\bar{x} \pm z s_{\bar{x}}$.

a. $n = 100$, $\bar{x} = 55.32$, $s = 8.4$, and $s_{\bar{x}} = s/\sqrt{n} = 8.4/\sqrt{100} = .84$

The 90% confidence interval for μ is $\bar{x} \pm z s_{\bar{x}} = 55.32 \pm 1.65(.84) = 55.32 \pm 1.39 = 53.93$ to 56.71

b. $n = 100$, $\bar{x} = 57.40$, $s = 7.5$, and $s_{\bar{x}} = s/\sqrt{n} = 7.5/\sqrt{100} = .75$

The 90% confidence interval for μ is $\bar{x} \pm z s_{\bar{x}} = 57.40 \pm 1.65(.75) = 57.40 \pm 1.24 = 56.16$ to 58.64

c. $n = 100$, $\bar{x} = 56.25$, $s = 7.9$, and $s_{\bar{x}} = s/\sqrt{n} = 7.9/\sqrt{100} = .79$

The 90% confidence interval for μ is $\bar{x} \pm z s_{\bar{x}} = 56.25 \pm 1.65(.79) = 56.25 \pm 1.30 = 54.95$ to 57.55

d. The confidence intervals of parts a and c cover μ but the confidence interval of part b does not.

8.51 In each of the following parts, since $n = 400$ is very large, we use the normal distribution to approximate the t distribution, and construct the confidence interval using $\bar{x} \pm z s_{\bar{x}}$.

a. $n = 400$, $\bar{x} = 92.45$, $s = 12.20$, and $s_{\bar{x}} = s/\sqrt{n} = 12.20/\sqrt{400} = .61$

The 98% confidence interval for μ is $\bar{x} \pm z s_{\bar{x}} = 92.45 \pm 2.33(.61) = 92.45 \pm 1.42 = 91.03$ to 93.87

b. $n = 400$, $\bar{x} = 91.75$, $s = 14.50$, and $s_{\bar{x}} = s/\sqrt{n} = 14.50/\sqrt{400} = .725$

The 98% confidence interval for μ is $\bar{x} \pm z s_{\bar{x}} = 91.75 \pm 2.33(.725) = 91.75 \pm 1.69 = 90.06$ to 93.44

c. $n = 400$, $\bar{x} = 89.63$, $s = 13.40$, and $s_{\bar{x}} = s/\sqrt{n} = 13.40/\sqrt{400} = .67$

The 98% confidence interval for μ is $\bar{x} \pm z s_{\bar{x}} = 89.63 \pm 2.33(.67) = 89.63 \pm 1.56 = 88.07$ to 91.19

d. The confidence intervals of parts b and c cover μ but the confidence interval of part a does not.

8.52 $n = 16$, $\bar{x} = 31$ minutes, $s = 7$ minutes, and $s_{\bar{x}} = s/\sqrt{n} = 7/\sqrt{16} = 1.75$ minutes

$df = n - 1 = 16 - 1 = 15$, $\alpha/2 = .5 - (.99/2) = .005$, and $t = 2.947$

The 99% confidence interval for μ is $\bar{x} \pm t s_{\bar{x}} = 31 \pm 2.947(1.75) = 31 \pm 5.16 = 25.84$ to 36.16 minutes

8.53 $n = 20$, $\bar{x} = 41.2$ bushels per acre, $s = 3$ bushels per acre, and $s_{\bar{x}} = s/\sqrt{n} = 3/\sqrt{20} = .67082039$ bushel per acre

$df = n - 1 = 20 - 1 = 19$, $\alpha/2 = .5 - (.90/2) = .05$, and $t = 1.729$

The 90% confidence interval for μ is

$\bar{x} \pm t s_{\bar{x}} = 41.2 \pm 1.729(.67082039) = 41.2 \pm 1.16 = 40.04$ to 42.36 bushels per acre

8.54 $n = 60$, $\bar{x} = 25.9$ years, $s = 3.2$ years, and $s_{\bar{x}} = s/\sqrt{n} = 3.2/\sqrt{60} = .41311822$ years

$df = n - 1 = 60 - 1 = 59$, $\alpha/2 = .5 - (.90/2) = .05$, and $t = 1.671$

The 90% confidence interval for μ is

$\bar{x} \pm t s_{\bar{x}} = 25.9 \pm 1.671(.41311822) = 25.9 \pm .69 = 25.21$ to 26.59 years

8.55 $n = 32$, $\bar{x} = .34$ gram, $s = .062$ gram, and $s_{\bar{x}} = s/\sqrt{n} = .062/\sqrt{32} = .01096016$ gram

$df = n - 1 = 32 - 1 = 31$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.040$

The 95% confidence interval for μ is $\bar{x} \pm t s_{\bar{x}} = .34 \pm 2.040(.01096016) = .34 \pm .02 = .32$ to $.36$ gram

8.56 $n = 25$, $\bar{x} = \$253$, $s = \$47$, and $s_{\bar{x}} = s/\sqrt{n} = 47/\sqrt{25} = \$9.40$

$df = n - 1 = 25 - 1 = 24$, $\alpha/2 = .5 - (.98/2) = .01$, and $t = 2.492$

The 98% confidence interval for μ is $\bar{x} \pm t s_{\bar{x}} = 253 \pm 2.492(9.40) = 253 \pm 23.42 = \$229.58$ to $\$276.42$

8.57 $n = 25$, $\bar{x} = 22$ minutes, $s = 6$ minutes, and $s_{\bar{x}} = s/\sqrt{n} = 6/\sqrt{25} = 1.2$ minutes

$df = n - 1 = 25 - 1 = 24$, $\alpha/2 = .5 - (.99/2) = .005$, and $t = 2.797$

The 99% confidence interval for μ is $\bar{x} \pm ts_{\bar{x}} = 22 \pm 2.797(1.2) = 22 \pm 3.36 = 18.64$ to 25.36 minutes

8.58 a. $n = 36$, $\bar{x} = 26.4$ mpg, $s = 2.3$ mpg, and $s_{\bar{x}} = s/\sqrt{n} = 2.3/\sqrt{36} = .38333333$ mpg

$df = n - 1 = 36 - 1 = 35$, $\alpha/2 = .5 - (.99/2) = .005$, and $t = 2.724$

The 99% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 26.4 \pm 2.724(.38333333) = 26.4 \pm 1.04 = 25.36$ to 27.44 mpg

b. The width of the confidence interval obtained in part a can be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate for μ.

8.59 a. $n = 40$, $\bar{x} = 23$ hours, $s = 3.75$ hours, and $s_{\bar{x}} = s/\sqrt{n} = 3.75/\sqrt{40} = .59292706$ hour

$df = n - 1 = 40 - 1 = 39$, $\alpha/2 = .5 - (.98/2) = .01$, and $t = 2.426$

The 98% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 23 \pm 2.426(.59292706) = 23 \pm 1.44 = 21.56$ to 24.44 hours

b. The width of the confidence interval obtained in part a can be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate for μ.

8.60 $n = 10$, $\sum x = 742$, and $\sum x^2 = 55{,}310$

$\bar{x} = (\sum x)/n = 742/10 = 74.20$ mph

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{55{,}310 - \frac{(742)^2}{10}}{10-1}} = 5.30827446 \text{ mph}$$

$s_{\bar{x}} = s/\sqrt{n} = 5.30827446/\sqrt{10} = 1.67862377$ mph

$df = n - 1 = 10 - 1 = 9$, $\alpha/2 = .5 - (.90/2) = .05$, and $t = 1.833$

The 90% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 74.20 \pm 1.833(1.67862377) = 74.20 \pm 3.08 = 71.12$ to 77.28 mph

8.61 $n = 9$, $\sum x = 72$, and $\sum x^2 = 708$

$\bar{x} = (\sum x)/n = 72/9 = 8$ hours

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{708 - \frac{(72)^2}{9}}{9-1}} = 4.06201920 \text{ hours}$$

$s_{\bar{x}} = s/\sqrt{n} = 4.06201920/\sqrt{9} = 1.35400640$ hours

$df = n - 1 = 9 - 1 = 8$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.306$

The 95% confidence interval for μ is $\bar{x} \pm ts_{\bar{x}} = 8 \pm 2.306(1.35400640) = 8 \pm 3.12 = 4.88 \text{ to } 11.12$ hours

8.62 $n = 8$, $\sum x = 102.38$, and $\sum x^2 = 1311.1012$

$\bar{x} = (\sum x)/n = 102.38/8 = 12.80$ seconds

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{1311.1012 - \frac{(102.38)^2}{8}}{8-1}} = .35720142 \text{ second}$$

$s_{\bar{x}} = s/\sqrt{n} = .35720142/\sqrt{8} = .12628978$ second

$df = n - 1 = 8 - 1 = 7$, $\alpha/2 = .5 - (.98/2) = .01$, and $t = 2.998$

The 98% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 12.80 \pm 2.998(.12628978) = 12.80 \pm .38 = 12.42 \text{ to } 13.18$ seconds

8.63 $n = 15$, $\sum x = 115$, and $\sum x^2 = 891.88$

$\bar{x} = (\sum x)/n = 115/15 = 7.67$ ounces

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{891.88 - \frac{(115)^2}{15}}{15-1}} = .85412166 \text{ ounce}$$

$s_{\bar{x}} = s/\sqrt{n} = .85412166/\sqrt{15} = .22053326$ ounce

$df = n - 1 = 15 - 1 = 14$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.145$

The 95% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 7.67 \pm 2.145(.22053326) = 7.67 \pm .47 = 7.20 \text{ to } 8.14$ ounces

8.64 $n = 12$, $\sum x = 33.36$, and $\sum x^2 = 96.0334$

a. $\bar{x} = (\sum x)/n = 33.36/12 = 2.78$ liters

b. $$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{96.0334 - \frac{(33.36)^2}{12}}{12-1}} = .54710810 \text{ liter}$$

$s_{\bar{x}} = s/\sqrt{n} = .54710810/\sqrt{12} = .15793650$ liter

$df = n - 1 = 12 - 1 = 11$, $\alpha/2 = .5 - (.98/2) = .01$, and $t = 2.718$

The 98% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 2.78 \pm 2.718(.15793650) = 2.78 \pm .43 = 2.35$ to 3.21 liters

8.65 Since $n = 400$ is very large, we use the normal distribution to approximate the t distribution, and construct the confidence interval using $\bar{x} \pm zs_{\bar{x}}$.

$n = 400$, $\bar{x} = 6.18$ years, $s = 2.87$ years, and $s_{\bar{x}} = s/\sqrt{n} = 2.87/\sqrt{400} = .1435$ year

a. $\bar{x} = 6.18$ years

b. The 98% confidence interval for μ is

$\bar{x} \pm zs_{\bar{x}} = 6.18 \pm 2.33(.1435) = 6.18 \pm .33 = 5.85$ to 6.51 years

8.66 To estimate the mean time, μ, taken by a cashier to serve customers at a supermarket (assuming that service times are normally distributed):

1. Take a random sample of 40 customers served by this cashier.
2. Collect the information on serving time for these customers.
3. Calculate the sample mean, the standard deviation, and $s_{\bar{x}}$.
4. Choose the confidence level and determine the t value for $df = n - 1 = 40 - 1 = 39$.
5. Construct the confidence interval for μ using $\bar{x} \pm ts_{\bar{x}}$.

8.67 To estimate the mean waiting time, μ (assuming that waiting times are normally distributed):

1. Take a random sample of 45 customers at the bank.
2. Record the waiting time for each of the 45 customers.
3. Calculate the sample mean, the standard deviation, and $s_{\bar{x}}$.
4. Choose the confidence level and determine the t value for $df = n - 1 = 45 - 1 = 44$.
5. Construct the confidence interval for μ using $\bar{x} \pm ts_{\bar{x}}$.

Section 8.5

8.68 The normal distribution will be used to make a confidence interval for the population proportion if the sampling distribution of the sample proportion is (approximately) normal. We know from Chapter 7 (Section 7.7.3) that the sampling distribution of the sample proportion is (approximately) normal if $np > 5$ and $nq > 5$. Thus, the normal distribution can be used to make a confidence interval for the population proportion if $np > 5$ and $nq > 5$. If p and q are unknown, the sample may be considered large if $n\hat{p} > 5$ and $n\hat{q} > 5$.

8.69 The sample proportion $\hat{p}$ is the **point estimator of *p***.

8.70 a. $n = 50$, $\hat{p} = .25$, $\hat{q} = 1 - \hat{p} = 1 - .25 = .75$, $n\hat{p} = (50)(.25) = 12.5$, and $n\hat{q} = (50)(.75) = 37.5$

Since $n\hat{p} > 5$ and $n\hat{q} > 5$, the sample size is large enough to use the normal distribution.

b. $n = 160$, $\hat{p} = .03$, $\hat{q} = 1 - \hat{p} = 1 - .03 = .97$, $n\hat{p} = (160)(.03) = 4.8$, and $n\hat{q} = (160)(.97) = 155.2$

Since $n\hat{p} < 5$, the sample size is not large enough to use the normal distribution.

c. $n = 400$, $\hat{p} = .65$, $\hat{q} = 1 - \hat{p} = 1 - .65 = .35$, $n\hat{p} = (400)(.65) = 260$, and $n\hat{q} = (400)(.35) = 140$

Since $n\hat{p} > 5$ and $n\hat{q} > 5$, the sample size is large enough to use the normal distribution.

d. $n = 75$, $\hat{p} = .06$, $\hat{q} = 1 - \hat{p} = 1 - .06 = .94$, $n\hat{p} = (75)(.06) = 4.5$, and $n\hat{q} = (75)(.94) = 70.5$

Since $n\hat{p} < 5$, the sample size is not large enough to use the normal distribution.

8.71 a. $n = 80$, $\hat{p} = .85$, $\hat{q} = 1 - \hat{p} = 1 - .85 = .15$, $n\hat{p} = (80)(.85) = 68$, and $n\hat{q} = (80)(.15) = 12$

Since $n\hat{p} > 5$ and $n\hat{q} > 5$, the sample size is large enough to use the normal distribution.

b. $n = 110$, $\hat{p} = .98$, $\hat{q} = 1 - \hat{p} = 1 - .98 = .02$, $n\hat{p} = (110)(.98) = 107.8$, and $n\hat{q} = (110)(.02) = 2.2$

Since $n\hat{q} < 5$, the sample size is not large enough to use the normal distribution.

c. $n = 35$, $\hat{p} = .4$, $\hat{q} = 1 - \hat{p} = 1 - .4 = .6$, $n\hat{p} = (35)(.4) = 14$, and $n\hat{q} = (35)(.6) = 21$

Since $n\hat{p} > 5$ and $n\hat{q} > 5$, the sample size is large enough to use the normal distribution.

d. $n = 200$, $\hat{p} = .08$, $\hat{q} = 1 - \hat{p} = 1 - .08 = .92$, $n\hat{p} = (200)(.08) = 16$, and $n\hat{q} = (200)(.92) = 184$

Since $n\hat{p} > 5$ and $n\hat{q} > 5$, the sample size is large enough to use the normal distribution.

8.72 a. $n = 300$, $\hat{p} = .63$, $\hat{q} = 1 - \hat{p} = 1 - .63 = .37$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.63)(.37)/300} = .02787472$

The 95% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .63 \pm 1.96(.02787472) = .63 \pm .055 = .575$ to $.685$

b. $n = 300$, $\hat{p} = .59$, $\hat{q} = 1 - \hat{p} = 1 - .59 = .41$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.59)(.41)/300} = .02839601$

The 95% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .59 \pm 1.96(.02839601) = .59 \pm .056 = .534$ to $.646$

c. $n = 300$, $\hat{p} = .67$, $\hat{q} = 1 - \hat{p} = 1 - .67 = .33$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.67)(.33)/300} = .02714774$

The 95% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .67 \pm 1.96(.02714774) = .67 \pm .053 = .617$ to $.723$

d. The confidence intervals of parts a and c cover p, but the confidence interval of part b does not.

8.73 a. $n = 1100$, $\hat{p} = .32$, $\hat{q} = 1 - \hat{p} = 1 - .32 = .68$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.32)(.68)/1100} = .01406479$

The 90% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .32 \pm 1.65(.01406479) = .32 \pm .023 = .297$ to $.343$

b. $n = 1100$, $\hat{p} = .36$, $\hat{q} = 1 - \hat{p} = 1 - .36 = .64$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.36)(.64)/1100} = .01447255$

The 90% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .36 \pm 1.65(.01447255) = .36 \pm .024 = .336$ to $.384$

c. $n = 1100$, $\hat{p} = .30$, $\hat{q} = 1 - \hat{p} = 1 - .30 = .70$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.30)(.70)/1100} = .01381699$

The 90% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .30 \pm 1.65(.01381699) = .30 \pm .023 = .277$ to $.323$

d. The confidence intervals of parts a and b cover p, but the confidence interval of part c does not.

8.74 $n = 200$, $\hat{p} = .91$, $\hat{q} = 1 - \hat{p} = 1 - .91 = .09$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.91)(.09)/200} = .02023611$

a. The 90% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .91 \pm 1.65(.02023611) = .91 \pm .033 = .877$ to $.943$

b. The 95% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .91 \pm 1.96(.02023611) = .91 \pm .040 = .870$ to $.950$

c. The 99% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .91 \pm 2.58(.02023611) = .91 \pm .052 = .858$ to $.962$

d. Yes, the width of the confidence intervals increases as the confidence level increases. This occurs because as the confidence level increases, the value of z increases.

8.75 $n = 200$, $\hat{p} = .27$, $\hat{q} = 1 - \hat{p} = 1 - .27 = .73$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.27)(.73)/200} = .03139267$

a. The 99% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .27 \pm 2.58(.03139267) = .27 \pm .081 = .189$ to $.351$

b. The 97% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .27 \pm 2.17(.03139267) = .27 \pm .068 = .202$ to $.338$

c. The 90% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .27 \pm 1.65(.03139267) = .27 \pm .052 = .218$ to $.322$

d. Yes, the width of the confidence intervals decreases as the confidence level decreases. This occurs because as the confidence level decreases, the value of z decreases.

8.76 $\hat{p} = .73$ and $\hat{q} = 1 - \hat{p} = 1 - .73 = .27$

a. $n = 100$ and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.73)(.27)/100} = .04439595$

The 99% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .73 \pm 2.58(.04439595) = .73 \pm .115 = .615$ to $.845$

b. $n = 600$ and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.73)(.27)/600} = .01812457$

The 99% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .73 \pm 2.58(.01812457) = .73 \pm .047 = .683$ to $.777$

c. $n = 1500$ and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.73)(.27)/1500} = .01146298$

The 99% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .73 \pm 2.58(.01146298) = .73 \pm .030 = .700$ to $.760$

d. Yes, the width of the confidence intervals decreases as the sample size increases. This occurs because increasing the sample size decreases the standard deviation of the sample proportion.

8.77 $\hat{p} = .31$ and $\hat{q} = 1 - \hat{p} = 1 - .31 = .69$

a. $n = 1200$ and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.31)(.69)/1200} = .01335103$

The 95% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .31 \pm 1.96(.01335103) = .31 \pm .026 = .284$ to $.336$

b. $n = 500$ and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.31)(.69)/500} = .02068333$

The 95% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .31 \pm 1.96(.02068333) = .31 \pm .041 = .269$ to $.351$

c. $n = 80$ and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.31)(.69)/80} = .05170832$

The 95% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .31 \pm 1.96(.05170832) = .31 \pm .101 = .209$ to $.411$

d. Yes, the width of the confidence intervals increases as the sample size decreases. This occurs because decreasing the sample size increases the standard deviation of the sample proportion.

8.78 a. $E = .035$, $\hat{p} = .29$, $\hat{q} = 1 - \hat{p} = 1 - .29 = .71$, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2\hat{p}\hat{q}}{E^2} = \frac{(2.58)^2(.29)(.71)}{(.035)^2} = 1118.82 \approx 1119$$

b. $E = .035$, $p = q = .50$ for the most conservative sample size, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(2.58)^2(.50)(.50)}{(.035)^2} = 1358.45 \approx 1359$$

8.79 a. $E = .045$, $\hat{p} = .53$, $\hat{q} = 1 - \hat{p} = 1 - .53 = .47$, and $z = 2.33$ for 98% confidence level

$$n = \frac{z^2\hat{p}\hat{q}}{E^2} = \frac{(2.33)^2(.53)(.47)}{(.045)^2} = 667.82 \approx 668$$

b. $E = .045$, $p = q = .50$ for the most conservative sample size, and $z = 2.33$ for 98% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(2.33)^2(.50)(.50)}{(.045)^2} = 670.23 \approx 671$$

8.80 a. $E = .025$, $p = q = .50$ for the most conservative sample size, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(1.96)^2(.50)(.50)}{(.025)^2} = 1536.64 \approx 1537$$

b. $E = .05$, $p = q = .50$ for the most conservative sample size, and $z = 1.65$ for 90% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(1.65)^2(.50)(.50)}{(.05)^2} = 272.25 \approx 273$$

c. $E = .015$, $p = q = .50$ for the most conservative sample size, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(2.58)^2(.50)(.50)}{(.015)^2} = 7396$$

8.81 a. $E = .025$, $\hat{p} = .16$, $\hat{q} = 1 - \hat{p} = 1 - .16 = .84$, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2 \hat{p}\hat{q}}{E^2} = \frac{(2.58)^2(.16)(.84)}{(.025)^2} = 1431.39 \approx 1432$$

b. $E = .05$, $\hat{p} = .85$, $\hat{q} = 1 - \hat{p} = 1 - .85 = .15$, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2 \hat{p}\hat{q}}{E^2} = \frac{(1.96)^2(.85)(.15)}{(.05)^2} = 195.92 \approx 196$$

c. $E = .015$, $\hat{p} = .97$, $\hat{q} = 1 - \hat{p} = 1 - .97 = .03$, and $z = 1.65$ for 90% confidence level

$$n = \frac{z^2 \hat{p}\hat{q}}{E^2} = \frac{(1.65)^2(.97)(.03)}{(.015)^2} = 352.11 \approx 353$$

8.82 $n = 175$, $\hat{p} = .40$, $\hat{q} = 1 - \hat{p} = 1 - .40 = .60$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.40)(.60)/175} = .03703280$

a. $\hat{p} = .40$

b. The 98% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .40 \pm 2.33(.03703280) = .40 \pm .086 = .314$ to $.486$

$E = z s_{\hat{p}} = 2.33(.03703280) = .086$

8.83 $n = 200$, $\hat{p} = 74/200 = .37$, $\hat{q} = 1 - \hat{p} = 1 - .37 = .63$, and

$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.37)(.63)/200} = .03413942$

a. The 98% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .37 \pm 2.33(.03413942) = .37 \pm .080 = .290$ to $.450$

The corresponding interval for the population percentage is 29% to 45%.

b. The width of the confidence interval constructed in part a may be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate of p.

8.84 $n = 1400$, $\hat{p} = .11$, $\hat{q} = 1 - \hat{p} = 1 - .11 = .89$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.11)(.89)/1400} = .00836233$

a. $\hat{p} = .11$

b. The 95% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .11 \pm 1.96(.00836233) = .11 \pm .016 = .094$ to $.126$

$E = z s_{\hat{p}} = 1.96(.00836233) = .016$

8.85 $n = 240$, $\hat{p} = 96/240 = .40$, $\hat{q} = 1 - \hat{p} = 1 - .40 = .60$, and

$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.40)(.60)/240} = .03162278$

a. $\hat{p} = .40$, so the point estimate for the population percentage is 40%

b. The 97% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .40 \pm 2.17(.03162278) = .40 \pm .069 = .331$ to $.469$

The corresponding interval for the population percentage is 33.1% to 46.9%.

8.86 $n = 50$, $\hat{p} = 35/50 = .70$, $\hat{q} = 1 - \hat{p} = 1 - .70 = .30$, and

$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.70)(.30)/50} = .06480741$

a. The 98% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .70 \pm 2.33(.06480741) = .70 \pm .151 = .549$ to $.851$

The corresponding interval for the population percentage is 54.9% to 85.1%.

$E = zs_{\hat{p}} = 2.33(.06480741) = .151$

b. The width of the confidence interval constructed in part a may be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate of p.

8.87 a. $n = 50$, $\hat{p} = 19/50 = .38$, $\hat{q} = 1 - \hat{p} = 1 - .38 = .62$, and

$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.38)(.62)/50} = .06864401$

The 99% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .38 \pm 2.58(.06864401) = .38 \pm .177 = .203$ to $.557$

The corresponding interval for the population percentage is 20.3% to 55.7%.

b. The width of the confidence interval constructed in part a may be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate of p.

8.88 $n = 1430$, $\hat{p} = .0257$, $\hat{q} = 1 - \hat{p} = 1 - .0257 = .9743$, and

$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.0257)(.9743)/1430} = .00418451$

a. The 95% confidence interval for p is

$\hat{p} \pm zs_{\hat{p}} = .0257 \pm 1.96(.00418451) = .0257 \pm .0082 = .0175$ to $.0339$

b. The width of the confidence interval constructed in part a may be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate of p.

8.89 $n = 855$, $\hat{p} = .147$, $\hat{q} = 1 - \hat{p} = 1 - .147 = .853$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.147)(.853)/855} = .01211017$

a. The 95% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .147 \pm 1.96(.01211017) = .147 \pm .024 = .123$ to $.171$

b. The sample proportion of .147 is an estimate of p based on a random sample. Because of sampling error, this estimate might differ from the true proportion p, so we make an interval estimate to allow for this uncertainty and sampling error.

8.90 $n = 15$, $\hat{p} = 9/15 = .60$, $\hat{q} = 1 - \hat{p} = 1 - .60 = .40$, and

$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.60)(.40)/15} = .12649111$

a. $\hat{p} = .60$

b. The 95% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .60 \pm 1.96(.12649111) = .60 \pm .248 = .352$ to $.848$

The corresponding interval for the population percentage is 35.2% to 84.8%.

8.91 $n = 24$, $\hat{p} = 8/24 = .333$, $\hat{q} = 1 - \hat{p} = 1 - .333 = .667$, and

$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.333)(.667)/24} = .09620096$

a. $\hat{p} = .333$

b. The 99% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .333 \pm 2.58(.09620096) = .333 \pm .248 = .085$ to $.581$

The corresponding interval for the population percentage is 8.5% to 58.1%.

8.92 $E = .02$, $p = q = .50$ for the most conservative sample size, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(2.58)^2(.50)(.50)}{(.02)^2} = 4160.25 \approx 4161$$

8.93 $E = .02$, $\hat{p} = .93$, $\hat{q} = 1 - \hat{p} = 1 - .93 = .07$, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2 \hat{p}\hat{q}}{E^2} = \frac{(2.58)^2(.93)(.07)}{(.02)^2} = 1083.33 \approx 1084$$

8.94 $E = .03$, $\hat{p} = .76$, $\hat{q} = 1 - \hat{p} = 1 - .76 = .24$, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2 \hat{p}\hat{q}}{E^2} = \frac{(2.58)^2(.76)(.24)}{(.03)^2} = 1349.03 \approx 1350$$

8.95 $E = .03$, $p = q = .50$ for the most conservative sample size, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(2.58)^2(.50)(.50)}{(.03)^2} = 1849$$

8.96 To estimate the proportion of students who hold off campus jobs:

1. Take a random sample of 40 students at your college.
2. Determine the number of students in this sample who hold off campus jobs.

3. Calculate $\hat{p}$ and $s_{\hat{p}}$
4. Choose the confidence level and find the required value of z in the normal distribution table.
5. Construct the confidence interval for p using the formula $\hat{p} \pm z s_{\hat{p}}$.

8.97 To estimate the percentage of students who are satisfied with campus food services:

1. Take a random sample of 30 students who use campus food services.
2. Determine the number of students in this sample who are satisfied with the campus food services.
3. Calculate $\hat{p}$ and $s_{\hat{p}}$.
4. Choose the confidence level and find the required value of z in the normal distribution table.
5. Construct the confidence interval for p using the formula $\hat{p} \pm z s_{\hat{p}}$.
6. Obtain the corresponding interval for the population percentage by multiplying the upper and lower limits of the confidence interval for p by 100%.

Supplementary Exercises

8.98 $n = 100$, $\bar{x} = \$273$, $\sigma = \$60$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 60/\sqrt{100} = \$6$

a. $\bar{x} = \$273$

b. The 95% confidence interval for μ is $\bar{x} \pm z\sigma_{\bar{x}} = 273 \pm 1.96(6) = 273 \pm 11.76 = \$261.24$ to $\$284.76$

8.99 $n = 100$, $\bar{x} = \$2640$, $\sigma = \$578$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 578/\sqrt{100} = \$57.80$

a. $\bar{x} = \$2640$

b. The 97% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 2640 \pm 2.17(57.80) = 2640 \pm 125.43 = \$2514.57$ to $\$2765.43$

8.100 $n = 25$, $\bar{x} = 24.015$ inches, $\sigma = .06$ inch, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .06/\sqrt{25} = .012$ inch

The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 24.015 \pm 2.58(.012) = 24.015 \pm .031 = 23.984$ to 24.046 inches

Since the upper limit of the confidence interval is 24.046, which is greater than 24.025, the machine needs an adjustment.

8.101 $n = 20$, $\bar{x} = 3.99$ inches, $\sigma = .04$ inch, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .04/\sqrt{20} = .00894427$ inch

The 98% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 3.99 \pm 2.33(.00894427) = 3.99 \pm .021 = 3.969$ to 4.011 inches

Since the lower limit of the confidence interval is 3.969, which is less than 3.98, the machine needs an adjustment.

8.102 $n = 35$, $\sum x = 1806$, and $\sum x^2 = 116{,}222$

$\bar{x} = (\sum x)/n = 1806/35 = 51.6$ minutes

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{116{,}222 - \frac{(1806)^2}{35}}{35-1}} = 26.02736117 \text{ minutes}$$

$s_{\bar{x}} = s/\sqrt{n} = 26.02736117/\sqrt{35} = 4.39942701$ minutes

$df = n - 1 = 35 - 1 = 34$, $\alpha/2 = .5 - (.99/2) = .005$, and $t = 2.728$

The 99% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 51.6 \pm 2.728(4.39942701) = 51.6 \pm 12.00 = 39.60$ to 63.60 minutes

8.103 $n = 44$, $\sum x = 637.3$, and $\sum x^2 = 11{,}038.55$

$\bar{x} = (\sum x)/n = 637.3/44 = 14.48$ gallons

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{11{,}038.55 - \frac{(637.3)^2}{44}}{44-1}} = 6.48403919 \text{ gallons}$$

$s_{\bar{x}} = s/\sqrt{n} = 6.48403919/\sqrt{44} = .97750569$ gallon

$df = n - 1 = 44 - 1 = 43$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.017$

The 95% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 14.48 \pm 2.017(.97750569) = 14.48 \pm 1.97 = 12.51$ to 16.45 gallons

8.104 $n = 25$, $\bar{x} = \$685$, $s = \$74$, and $s_{\bar{x}} = s/\sqrt{n} = 74/\sqrt{25} = \$14.80$

$df = n - 1 = 25 - 1 = 24$, $\alpha/2 = .5 - (.99/2) = .005$, and $t = 2.797$

The 99% confidence interval for μ is $\bar{x} \pm ts_{\bar{x}} = 685 \pm 2.797(14.80) = 685 \pm 41.40 = \$643.60$ to $\$726.40$

8.105 $n = 18$, $\bar{x} = 24$ minutes, $s = 4.5$ minutes, and $s_{\bar{x}} = s/\sqrt{n} = 4.5/\sqrt{18} = 1.06066017$ minutes

$df = n - 1 = 18 - 1 = 17$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.110$

The 95% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 24 \pm 2.110(1.06066017) = 24 \pm 2.24 = 21.76$ to 26.24 minutes

8.106 Since $n = 500$ is very large, we use the normal distribution to approximate the t distribution, and construct the confidence interval using $\bar{x} \pm zs_{\bar{x}}$.

$n = 500$, $\bar{x} = 9.75$ hours, $s = 2.2$ hours, and $s_{\bar{x}} = s/\sqrt{n} = 2.2/\sqrt{500} = .09838699$ hour

The 90% confidence interval for μ is $\bar{x} \pm zs_{\bar{x}} = 9.75 \pm 1.65(.09838699) = 9.75 \pm .16 = 9.59$ to 9.91 hours

8.107 Since $n = 300$ is very large, we use the normal distribution to approximate the t distribution, and construct the confidence interval using $\bar{x} \pm zs_{\bar{x}}$.

$n = 300$, $\bar{x} = 4.5$ hours, $s = .75$ hour, and $s_{\bar{x}} = s\big/\sqrt{n} = .75\big/\sqrt{300} = .04330127$ hour

The 98% confidence interval for μ is $\bar{x} \pm zs_{\bar{x}} = 4.5 \pm 2.33(.04330127) = 4.5 \pm .10 = 4.40$ to 4.60 hours

8.108 $n = 12$, $\sum x = 26.80$, and $\sum x^2 = 62.395$

$\bar{x} = (\sum x)/n = 26.80/12 = 2.23$ hours

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{62.395 - \frac{(26.80)^2}{12}}{12-1}} = .48068764 \text{ hour}$$

$s_{\bar{x}} = s\big/\sqrt{n} = .48068764\big/\sqrt{12} = .13876257$ hour

$df = n - 1 = 12 - 1 = 11$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.201$

The 95% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 2.23 \pm 2.201(.13876257) = 2.23 \pm .31 = 1.92$ to 2.54 hours

8.109 $n = 10$, $\sum x = 1514$, and $\sum x^2 = 229{,}646$

$\bar{x} = (\sum x)/n = 1514/10 = 151.40$ calories

$$s = \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} = \sqrt{\frac{229{,}646 - \frac{(1514)^2}{10}}{10-1}} = 6.88315173 \text{ calories}$$

$s_{\bar{x}} = s\big/\sqrt{n} = 6.88315173\big/\sqrt{10} = 2.17664369$ calories

$df = n - 1 = 10 - 1 = 9$, $\alpha/2 = .5 - (.99/2) = .005$, and $t = 3.250$

The 99% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 151.40 \pm 3.250(2.17664369) = 151.40 \pm 7.07 = 144.33$ to 158.47 calories

8.110 $n = 50$, $\hat{p} = .12$, $\hat{q} = 1 - \hat{p} = 1 - .12 = .88$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.12)(.88)/50} = .04595650$

a. $\hat{p} = .12$, so the point estimate for the population percentage is 12%

b. The 99% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .12 \pm 2.58(.04595650) = .12 \pm .119 = .001$ to $.239$

The corresponding interval for the population percentage is 0.1% to 23.9%.

8.111 $n = 430$, $\hat{p} = .033$, $\hat{q} = 1 - \hat{p} = 1 - .033 = .967$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.033)(.967)/430} = .00861462$

a. $\hat{p} = .033$

b. The 95% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .033 \pm 1.96(.00861462) = .033 \pm .017 = .016$ to $.050$

c. The player's suspicion seems reasonable since .0526 falls outside the 95% confidence interval.

8.112 $n = 20$, $\hat{p} = 8/20 = .40$, $\hat{q} = 1 - \hat{p} = 1 - .40 = .60$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.40)(.60)/20} = .10954451$

The 99% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .40 \pm 2.58(.10954451) = .40 \pm .283 = .117$ to $.683$

The corresponding interval for the population percentage is 11.7% to 68.3%.

8.113 $n = 16$, $\hat{p} = 5/16 = .313$, $\hat{q} = 1 - \hat{p} = 1 - .313 = .687$, and

$$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.313)(.687)/16} = .11592859$$

The 97% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .313 \pm 2.17(.11592859) = .313 \pm .252 = .061$ to $.565$

The corresponding interval for the population percentage is 6.1% to 56.5%.

8.114 $E = 1.2$ hours, $\sigma = 3$ hours, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.58)^2(3)^2}{(1.2)^2} = 41.60 \approx 42$$

8.115 $E = \$3500$, $\sigma = \$31{,}500$, and $z = 1.65$ for 90% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.65)^2(31{,}500)^2}{(3500)^2} = 220.52 \approx 221$$

8.116 $E = .05$, $p = q = .50$ for the most conservative sample size, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(1.96)^2(.50)(.50)}{(.05)^2} = 384.16 \approx 385$$

8.117 $E = .05$, $\hat{p} = .63$, $\hat{q} = 1 - \hat{p} = 1 - .63 = .37$, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2 \hat{p}\hat{q}}{E^2} = \frac{(1.96)^2(.63)(.37)}{(.05)^2} = 358.19 \approx 359$$

8.118 $n > 30$; The 95% confidence interval for μ, based on a large sample, is $\bar{x} \pm 1.96\sigma_{\bar{x}} = \$8.46$ to $\$9.86$.

a. $\bar{x} = \dfrac{\$8.46 + \$9.86}{2} = \$9.16$

b. Since $\bar{x} + 1.96\,\sigma_{\bar{x}} = 9.86$, $\sigma_{\bar{x}} = \dfrac{9.86 - \bar{x}}{1.96} = \dfrac{9.86 - 9.16}{1.96} = .35714286$

The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 9.16 \pm 2.58(.35714286) = 9.16 \pm .92 = \$8.24$ to $\$10.08$

8.119 Let p_1 = proportion of the population who “cut back on vacations or entertainment because of their cost,” p_2 = proportion of population who “failed to pay a bill on time,” and p_3 = proportion of the population who “have not gone to a doctor because of the cost.” Note that for $n = 1004$, $n\hat{p}$ and $n\hat{q}$ exceed 5 for all of these proportions, so the sample is considered large.

$\hat{p}_1 = .64$, $\hat{q}_1 = 1 - \hat{p}_1 = 1 - .64 = .36$, and $s_{\hat{p}_1} = \sqrt{\hat{p}_1\hat{q}_1 / n} = \sqrt{(.64)(.36)/1004} = .01514867$

The 95% confidence interval for p_1 is $\hat{p}_1 \pm z s_{\hat{p}_1} = .64 \pm 1.96(.01514867) = .64 \pm .030 = .610$ to $.670$

The corresponding interval for the population percentage is 61.0% to 67.0%.

$\hat{p}_2 = .37$, $\hat{q}_2 = 1 - \hat{p}_2 = 1 - .37 = .63$, and $s_{\hat{p}_2} = \sqrt{\hat{p}_2\hat{q}_2 / n} = \sqrt{(.37)(.63)/1004} = .01523717$

The 95% confidence interval for p_2 is $\hat{p}_2 \pm z s_{\hat{p}_2} = .37 \pm 1.96(.01523717) = .37 \pm .030 = .340$ to $.400$

The corresponding interval for the population percentage is 34.0% to 40.0%.

$\hat{p}_3 = .25$, $\hat{q}_3 = 1 - \hat{p}_3 = 1 - .25 = .75$, and $s_{\hat{p}_3} = \sqrt{\hat{p}_3\hat{q}_3 / n} = \sqrt{(.25)(.75)/1004} = .01366576$

The 95% confidence interval for p_3 is $\hat{p}_3 \pm z s_{\hat{p}_3} = .25 \pm 1.96(.01366576) = .25 \pm .027 = .223$ to $.277$

The corresponding interval for the population percentage is 22.3% to 27.7%.

(1) A confidence interval is a range of numbers (in this particular case proportions or percentages) which gives an estimate for the true value of the population parameter. (2) A single percentage that we assign as an estimate would almost always differ from the true value, hence a range with the associated confidence level is more informative. (3) The 95% means that we are 95% confident that this interval, calculated from this particular sample, actually contains the true value of the population parameter. (4) We assume that the 1004 persons selected for the sample constitute a random sample of adults.

8.120 $n = 100$, $\hat{p} = 10/100 = .10$, $\hat{q} = 1 - \hat{p} = 1 - .10 = .90$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.10)(.90)/100} = .03$

a. The 95% confidence interval for p is $\hat{p} \pm z s_{\hat{p}} = .10 \pm 1.96(.03) = .10 \pm .059 = .041$ to $.159$

b. The corresponding interval for the population percentage is 4.1% to 15.9%, and 18% does not lie within this interval. This suggests that the vaccine is effective to some degree.

c. Answers will vary. One factor which may have distorted the outcome of this experiment is that the owners who agreed to participate in this experiment may be more concerned about their dogs' health and restrict the area that the dogs can access, thereby decreasing the exposure to the ticks.

8.121 $E = 1.0$ mile, $s = 4.1$ miles, and $z = 1.96$ for 95% confidence level; σ may be estimated by s

$$n = \frac{z^2 s^2}{E^2} = \frac{(1.96)^2(4.1)^2}{(1.0)^2} = 64.58 \approx 65$$

Thus, an additional 65 – 20 = 45 observations must be taken.

8.122 The major problem with this procedure is that the sample is not drawn from the whole target population. This introduces nonsampling error.

1) Households with no cars would be excluded from this sample. Furthermore, households with more than 2 cars could be counted more than once in the sample. Both of these problems would result in an upwardly biased estimate of p.

2) Because of the characteristics of this particular gas station, the clientele may not be typical of all gas station clientele (with respect to multiple car ownership). Other problems may be present, such as the next 200 gasoline customers may not be typical of all customers, or there may be dependency between gasoline customers from the same household. If the attendant took a random sample of 200 households, and determined how many of these households owned more than 2 cars, the sampling error would still be present, but could be reduced by taking a larger sample.

8.123 a. $E = 100$ cars, $\sigma = 170$ cars, and $z = 2.58$ for 99% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(2.58)^2(170)^2}{(100)^2} = 19.24 \approx 20$$

Note that since $n < 30$, we must assume that the number of cars passing each day is approximately normally distributed, or we may take a larger $(n \geq 30)$ sample.

b. Since $n = \dfrac{z^2\sigma^2}{E^2}$, $z = \dfrac{E\sqrt{n}}{\sigma} = \dfrac{100\sqrt{20}}{272} = 1.64$ which corresponds to a confidence level of approximately 90%.

c. Since $n = \dfrac{z^2\sigma^2}{E^2}$, $E = \dfrac{z\sigma}{\sqrt{n}} = \dfrac{(2.58)(130)}{\sqrt{20}} = 75$.

Thus, they can be 99% confident that their point estimate is within 75 cars of the true average.

8.124 No, the student's analysis does not make sense. The relevant parameter, p, is the proportion of all U.S. senators in favor of the bill. The value .55 is not a sample proportion; instead it is the population proportion, p. Since $p = .55$ is known there is no need to estimate it.

8.125 When the sample size is doubled, the margin of error is reduced by a factor of $\sqrt{2}/2$. When the sample size is quadrupled, the margin of error is reduced by a factor of 1/2. This relationship does not hold true when calculating a confidence interval for the population mean with an unknown population standard deviation for the following reasons:

1) While σ is constant, the sample standard deviation, s, is not. The sample standard deviation will change with each sample.
2) If σ is unknown, the confidence interval is calculated with a t value. This value will change as the sample size changes because the degrees of freedom will change.

8.126 Since $E = z\sigma_{\bar{x}} = z\left(\sigma/\sqrt{n}\right)$, if we select a smaller sample than required for a specific confidence level, the margin of error will be larger than desired.

8.127 Answers will vary. The following provide one possible example for each case.

a. $n = 200$, $\hat{p} = .01$, $\hat{q} = 1 - \hat{p} = 1 - .01 = .99$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.01)(.99)/200} = .00703562$

The 95% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .01 \pm 1.96(.00703562) = .01 \pm .014 = -.004$ to $.024$

b. $n = 160$, $\hat{p} = .9875$, $\hat{q} = 1 - \hat{p} = 1 - .9875 = .0125$, and

$s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.9875)(.0125)/160} = .00878342$

The 95% confidence interval for p is

$\hat{p} \pm zs_{\hat{p}} = .9875 \pm 1.96(.00878342) = .9875 \pm .0172 = .9703$ to 1.0047

The value of the lower limit in the first interval is less than 0, and the value of the upper limit in the second interval is greater than 1. These are not possible values for proportions.

Self – Review Test

1. a. Estimation means assigning values to a *population parameter* based on the value of a *sample statistic*.

b. An estimator is the *sample statistic* used to estimate a *population parameter*.

c. The value of a *sample statistic* is called the point estimate of the corresponding *population parameter*.

2. b **3.** a **4.** a **5.** c **6.** b

7. $n = 36$, $\bar{x} = \$159{,}000$, $\sigma = \$27{,}000$, and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 27{,}000/\sqrt{36} = \$4500$

a. $\bar{x} = \$159{,}000$

b. The 99% confidence interval for μ is

$\bar{x} \pm z\sigma_{\bar{x}} = 159{,}000 \pm 2.58(4500) = 159{,}000 \pm 11{,}610 = \$147{,}390$ to $\$170{,}610$

$E = z\sigma_{\bar{x}} = 2.58(4500) = \$11{,}610$

8. $n = 25$, $\bar{x} = \$410{,}425$, $s = \$74{,}820$, and $s_{\bar{x}} = s/\sqrt{n} = 74{,}820/\sqrt{25} = \$14{,}964$

$df = n - 1 = 25 - 1 = 24$, $\alpha/2 = .5 - (.95/2) = .025$, and $t = 2.064$

The 95% confidence interval for μ is

$\bar{x} \pm ts_{\bar{x}} = 410{,}425 \pm 2.064(14{,}964) = 410{,}425 \pm 30{,}885.70 = \$379{,}539.30$ to $\$441{,}310.70$

9. $n = 450$, $\hat{p} = .55$, $\hat{q} = 1 - \hat{p} = 1 - .55 = .45$, and $s_{\hat{p}} = \sqrt{\hat{p}\hat{q}/n} = \sqrt{(.55)(.45)/450} = .02345208$

a. $\hat{p} = .55$

b. The 99% confidence interval for p is $\hat{p} \pm zs_{\hat{p}} = .55 \pm 2.58(.02345208) = .55 \pm .061 = .489$ to $.611$

10. $E = 1.25$ degrees, $\sigma = 5.78$ degrees, and $z = 1.96$ for 95% confidence level

$$n = \frac{z^2\sigma^2}{E^2} = \frac{(1.96)^2(5.78)^2}{(1.25)^2} = 82.14 \approx 83$$

11. $E = .05$, $p = q = .50$ for the most conservative sample size, and $z = 1.65$ for 90% confidence level

$$n = \frac{z^2 pq}{E^2} = \frac{(1.65)^2(.50)(.50)}{(.05)^2} = 272.25 \approx 273$$

12. $E = .05$, $\hat{p} = .70$, $\hat{q} = 1 - \hat{p} = 1 - .70 = .30$, and $z = 1.65$ for 90% confidence level

$$n = \frac{z^2 \hat{p}\hat{q}}{E^2} = \frac{(1.65)^2(.70)(.30)}{(.05)^2} = 228.69 \approx 229$$

13. The width of the confidence interval can be reduced by:

1. Lowering the confidence level
2. Increasing the sample size

The second alternative is better because lowering the confidence level results in a less reliable estimate for μ..

14. To estimate the mean number of hours that all students at your college work per week:

1. Take a random sample of 12 students from your college. (Note that if you want to estimate the mean number of hours that all students at your college *who hold jobs* work per week, you would limit your sample to students who hold jobs.)
2. Record the number of hours each of these students worked last week.
3. Calculate $\bar{x}$, s and $s_{\bar{x}}$ from these data.
4. After choosing the confidence level, find the value for the t distribution with 11 *df* and for an area of $\alpha/2$ in the right tail.
5. Construct the confidence interval for μ by using the formula $\bar{x} \pm ts_{\bar{x}}$.

   You are assuming that the hours worked by all students at your college have a normal distribution.

15. To estimate the proportion of people who are happy with their current jobs:

1. Take a random sample of 35 workers.
2. Determine whether or not each worker is happy with his or her job.
3. Calculate $\hat{p}$, $\hat{q}$, and $s_{\hat{p}}$.
4. Choose the confidence level and find the required value of z from the normal distribution table.
5. Construct the confidence interval for p by using the formula $\hat{p} \pm zs_{\hat{p}}$.