Chapter Seven

Sections 7.1 - 7.2

7.1 The probability distribution of the population data is called the **population distribution**. Table 7.2 on p. 301 of the text provides an example of such a distribution. The probability distribution of a sample statistic is called its **sampling distribution**. Table 7.5 on p. 303 of the text provides an example of the sampling distribution of the sample mean.

7.2 **Sampling error** is the difference between the value of the sample statistic and the value of the corresponding population parameter, assuming that the sample is random and no nonsampling error has been made. Example 7–1 on pp. 304-305 of the text displays sampling error. Sampling error occurs only in sample surveys.

7.3 **Nonsampling errors** are errors that may occur in the collection, recording, and tabulation of data. Example 7–1 on pp. 304-305 of the text exhibits nonsampling error. Nonsampling errors can occur in both a sample survey and a census.

7.4 a. $\mu = (15 + 13 + 8 + 17 + 9 + 12)/6 = 74/6 = 12.33$

b. $\bar{x} = (13 + 8 + 9 + 12)/4 = 42/4 = 10.50$

Sampling error $= \bar{x} - \mu = 10.50 - 12.33 = -1.83$

c. Liza's incorrect $\bar{x} = (13 + 8 + 6 + 12)/4 = 39/4 = 9.75$

$\bar{x} - \mu = 9.75 - 12.33 = -2.58$

Sampling error (from part b) = –1.83

Nonsampling error = $-2.58 - (-1.83) = -.75$

d.

Sample	$\bar{x}$	$\bar{x}-\mu$
15, 13, 8, 17	13.25	.92
15, 13, 8, 9	11.25	–1.08
15, 13, 17, 9	13.50	1.17
15, 8, 17, 9	12.25	–.08
13, 8, 17, 9	11.75	–.58
15, 13, 8, 12	12.00	–.33
15, 13, 17, 12	14.25	1.92
15, 8, 17, 12	13.00	.67
13, 8, 17, 12	12.50	.17
15, 13, 9, 12	12.25	–.08
15, 8, 9, 12	11.00	–1.33
13, 8, 9, 12	10.50	–1.83
15, 17, 9, 12	13.25	.92
13, 17, 9, 12	12.75	.42
8, 17, 9, 12	11.50	–.83

7.5 a. $\mu = (20 + 25 + 13 + 19 + 9 + 15 + 11 + 7 + 17 + 30)/10 = 166/10 = 16.60$

b. $\bar{x} = (20 + 25 + 13 + 9 + 15 + 11 + 7 + 17 + 30)/9 = 147/9 = 16.33$

Sampling error = $\bar{x} - \mu = 16.33 - 16.60 = -.27$

c. Rich's incorrect $\bar{x} = (20 + 25 + 13 + 9 + 15 + 11 + 17 + 17 + 30)/9 = 157/9 = 17.44$

$\bar{x} - \mu = 17.44 - 16.60 = .84$

Sampling error (from part b) = –.27

Nonsampling error = .84 – (–.27) = 1.11

d.

Sample	$\bar{x}$	$\bar{x}-\mu$
25, 13 19, 9,15,11,7,17,30	16.22	–.38
20, 13, 19, 9, 15, 11, 7, 17, 30	15.67	–.93
20, 25 19, 9, 15, 11, 7, 17, 30	17.00	.40
20, 25, 13, 9, 15, 11, 7, 17, 30	16.33	–.27
20, 25, 13, 19, 15, 11, 7, 17, 30	17.44	.84
20, 25, 13, 19, 9, 11, 7, 17, 30	16.78	.18
20, 25, 13, 19, 9, 15, 7, 17, 30	17.22	.62
20, 25, 13, 19, 9, 15, 11, 17, 30	17.67	1.07
20, 25, 13, 19, 9, 15, 11, 7, 30	16.56	–.04
20, 25, 13, 19, 9, 15, 11, 7, 17	15.11	–1.49

7.6

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
70	.20	14.00	4900	980.00
78	.20	15.60	6084	1216.80
80	.40	32.00	6400	2560.00
95	.20	19.00	9025	1805.00
		$\Sigma xP(x) = 80.60$		$\Sigma x^2P(x) = 6561.80$

$\mu = \Sigma xP(x) = 80.60$

$$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{6561.80 - (80.60)^2} = 8.09$$

7.7 a.

x	$P(x)$
15	1/6=.167
21	1/6=.167
25	1/6=.167
28	1/6=.167
53	1/6=.167
55	1/6=.167

b.

Sample	$\bar{x}$
55, 53, 28, 25, 21	36.4
55, 53, 28, 25, 15	35.2
55, 53, 28, 21, 15	34.4
55, 53, 25, 21, 15	33.8
55, 28, 25, 21, 15	28.8
53, 28, 25, 21, 15	28.4

$\bar{x}$	$P(\bar{x})$
28.4	1/6=.167
28.8	1/6=.167
33.8	1/6=.167
34.4	1/6=.167
35.2	1/6=.167
36.4	1/6=.167

c. The mean for the population data is $\mu = (55 + 53 + 28 + 25 + 21 + 15)/6 = 197/6 = 32.83$

Suppose the random sample of five family members includes the observations: 55, 28, 25, 21, and 15. The mean for this sample is $\bar{x} = (55 + 28 + 25 + 21 + 15)/5 = 144/5 = 28.80$

Sampling error = $\bar{x} - \mu = 28.80 - 32.83 = -4.03$

7.8 a.

x	$P(x)$
7	2/5 = .40
8	1/5 = .20
14	1/5 = .20
20	1/5 = .20

b.

Sample	$\bar{x}$
14, 8, 7, 7	9.00
14, 8, 7, 20	12.25
14, 8, 7, 20	12.25
14, 7, 7, 20	12.00
8, 7, 7, 20	10.50

$\bar{x}$	$P(\bar{x})$
9.00	1/5 = .20
10.50	1/5 = .20
12.00	1/5 = .20
12.25	2/5 = .40

c. The mean for the population data is $\mu = (7 + 8 + 14 + 7 + 20)/5 = 56/5 = 11.20$

d. Suppose the random sample of four faculty members includes the observations: 14, 8, 7 and 20.
The mean for this sample is $\bar{x} = (14 + 8 + 7 + 20)/4 = 49/4 = 12.25$
Sampling error = $\bar{x} - \mu = 12.25 - 11.20 = 1.05$

Section 7.3

7.9 a. Mean of $\bar{x} = \mu_{\bar{x}} = \mu$

b. Standard deviation of $\bar{x}$ = $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ where σ = population standard deviation and n = sample size.

7.10 A sample statistic used to estimate a population parameter is called an **estimator**. An estimator is **unbiased** when its expected value is equal to the value of the corresponding population parameter. The sample mean $\bar{x}$ is an unbiased estimator of μ, because the mean of $\bar{x}$ is equal to μ.

7.11 An estimator is **consistent** when its standard deviation decreases as the sample size is increased. The sample mean $\bar{x}$ is a consistent estimator of μ because its standard deviation decreases as the sample size increases. As n increases, $\sqrt{n}$ increases, and, consequently, the value of $\sigma_{\bar{x}} = \sigma/\sqrt{n}$ decreases.

7.12 Since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, as n increases, $\sigma_{\bar{x}}$ decreases.

7.13 $\mu = 60$ and $\sigma = 10$

a. $\mu_{\bar{x}} = \mu = 60$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{18} = 2.357$

b. $\mu_{\bar{x}} = \mu = 60$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{90} = 1.054$

7.14 $\mu = 90$ and $\sigma = 18$

a. $\mu_{\bar{x}} = \mu = 90$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{10} = 5.692$

b. $\mu_{\bar{x}} = \mu = 90$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{35} = 3.043$

7.15 a. $n/N = 300/5000 = .06 > .05$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{25}{\sqrt{300}}\sqrt{\frac{5000-300}{5000-1}} = 1.400$$

b. Since $n/N = 100/5000 = .02 < .05$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 25/\sqrt{100} = 2.500$

7.16 a. Since $n/N = 2500/100{,}000 = .025 < .05$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 40/\sqrt{2500} = .800$

b. $n/N = 7000/100{,}000 = .07 > .05$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{40}{\sqrt{7000}}\sqrt{\frac{100{,}000-7000}{100{,}000-1}} = .461$$

7.17 $\mu = 125$ and $\sigma = 36$

a. Since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, $n = (\sigma/\sigma_{\bar{x}})^2 = (36/3.6)^2 = 100$

b. $n = (\sigma/\sigma_{\bar{x}})^2 = (36/2.25)^2 = 256$

7.18 $\mu = 46$ and $\sigma = 10$

a. Since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, $n = (\sigma/\sigma_{\bar{x}})^2 = (10/2.0)^2 = 25$

b. $n = (\sigma/\sigma_{\bar{x}})^2 = (10/1.6)^2 = 39$ approximately

7.19 $\mu = \$3.084$, $\sigma = \$.263$, and $n = 47$

$\mu_{\bar{x}} = \mu = \$3.084$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .263/\sqrt{47} = \$.038$

7.20 $\mu = 2300$ square feet, $\sigma = 500$ square feet, and $n = 25$

$\mu_{\bar{x}} = \mu = 2{,}300$ square feet and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 500/\sqrt{25} = 100$ square feet

7.21 $\mu = \$520$, $\sigma = \$72$, and $n = 25$

$\mu_{\bar{x}} = \mu = \$520$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 72/\sqrt{25} = \$14.40$

7.22 $\mu = \$55$, $\sigma = \$13.25$, and $n = 33$

$\mu_{\bar{x}} = \mu = \$55$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 13.25/\sqrt{33} = \$2.31$

7.23 $\sigma = \$2000$ and $\sigma_{\bar{x}} = \$125$

$n = (\sigma/\sigma_{\bar{x}})^2 = (2000/125)^2 = 256$ players

7.24 $\sigma = \$139.50$ million and $\sigma_{\bar{x}} = \$15.50$ million

$n = (\sigma/\sigma_{\bar{x}})^2 = (139.50/15.50)^2 = 81$

7.25 a.

$\bar{x}$	$P(\bar{x})$	$\bar{x}P(\bar{x})$	$\bar{x}^2$	$\bar{x}^2P(\bar{x})$
76.00	.20	15.200	5776.0000	1155.200
76.67	.10	7.667	5878.2889	587.829
79.33	.10	7.933	6293.2489	629.325
81.00	.10	8.100	6561.0000	656.100
81.67	.20	16.334	6669.9889	1333.998
84.33	.20	16.866	7111.5489	1422.310
85.00	.10	8.500	7225.0000	722.500
		$\sum\bar{x}P(\bar{x}) = 80.60$		$\sum\bar{x}^2P(\bar{x}) = 6507.262$

$\sum\bar{x}P(\bar{x}) = 80.60$ is the same value found in Exercise 7.6 for μ.

b. $\sigma_{\bar{x}} = \sqrt{\sum \bar{x}^2 P(\bar{x}) - \mu_{\bar{x}}^2} = \sqrt{6507.262 - (80.60)^2} = 3.302$

c. $\sigma/\sqrt{n} = 8.09/\sqrt{3} = 4.67$ is not equal to $\sigma_{\bar{x}} = 3.30$ in this case because $n/N = 3/5 = .60 > .05$.

d. $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}\sqrt{\frac{N-n}{N-1}} = \frac{8.09}{\sqrt{3}}\sqrt{\frac{5-3}{5-1}} = 3.302$

Section 7.4

7.26 The population from which the sample is drawn must be normally distributed.

7.27 The **central limit theorem** states that for a large sample, the sampling distribution of the sample mean is approximately normal, irrespective of the shape of the population distribution. Furthermore, $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, where μ and σ are the population mean and standard deviation, respectively. A sample size of 30 or more is considered large enough to apply the central limit theorem to $\bar{x}$.

7.28 The central limit theorem will apply in cases a and c since $n \geq 30$. It will not apply in case b because $n < 30$.

7.29
a. Slightly skewed to the right
b Approximately normal because $n \geq 30$ and the central limit theorem applies
c. Close to normal with a slight skew to the right

7.30 a. and b. In both cases the sampling distribution of $\bar{x}$ would be normal because the population distribution is normal.

7.31 a. and b. In both cases the sampling distribution of $\bar{x}$ would be normal because the population distribution is normal.

7.32 $\mu = 7.7$ minutes, $\sigma = 2.1$ minutes, and $n = 16$

$\mu_{\bar{x}} = \mu = 7.7$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.1/\sqrt{16} = .525$ minute

The sampling distribution of $\bar{x}$ is normal because the population is normally distributed.

7.33 $\mu = 20.20$ hours, $\sigma = 2.60$ hours, and $n = 18$

$\mu_{\bar{x}} = \mu = 20.20$ hours and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.60/\sqrt{18} = .613$ hour

The sampling distribution of $\bar{x}$ is approximately normal because the population is approximately normally distributed.

7.34 $\mu = \$140$, $\sigma = \$30$, and $n = 25$

$\mu_{\bar{x}} = \mu = \$140$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 30/\sqrt{25} = \$6$

The sampling distribution of $\bar{x}$ is approximately normal because the population is approximately normally distributed.

7.35 $\mu = 3.02$, $\sigma = .29$, $N = 5540$ and $n = 48$

$\mu_{\bar{x}} = \mu = 3.02$

Since $n/N = 48/5540 = .009 < .05$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .29/\sqrt{48} = .042$.

The sampling distribution of $\bar{x}$ is approximately normal because the population is approximately normally distributed.

7.36 $\mu = 133$ pounds, $\sigma = 24$ pounds, and $n = 45$

$\mu_{\bar{x}} = \mu = 133$ pounds and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 24/\sqrt{45} = 3.578$ pounds

The sampling distribution of $\bar{x}$ is approximately normal because the sample size is large ($n \geq 30$).

7.37 $\mu = 91.4$ grams, $\sigma = 93.25$ grams

For $n = 20$, $\mu_{\bar{x}} = \mu = 91.4$ grams and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 93.25/\sqrt{20} = 20.851$ grams

The sampling distribution of $\bar{x}$ is skewed to the right because the distribution of x is strongly skewed to the right and the sample size is not large ($n < 30$).

For $n = 75$, $\mu_{\bar{x}} = \mu = 91.4$ grams and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 93.25/\sqrt{75} = 10.768$ grams

The sampling distribution of $\bar{x}$ is approximately normal because the sample size is large ($n \geq 30$).

7.38 $\mu = \$78{,}000$, $\sigma = \$8300$, and $n = 50$

$\mu_{\bar{x}} = \mu = \$78{,}000$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 8300/\sqrt{50} = \$1173.80$

The sampling distribution of $\bar{x}$ is normal because the population distribution is normal.

7.39 $\mu = 200$ pieces, $\sigma = 145$ pieces, and $n = 84$

$\mu_{\bar{x}} = \mu = 200$ pieces and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 145/\sqrt{84} = 15.821$ pieces

The sampling distribution of $\bar{x}$ is approximately normal. We do not need to know the shape of the population distribution in order to make this conclusion because the sample size is large ($n \geq 30$) and the central limit theorem applies.

Section 7.5

7.40 $P(\mu - 2.50\sigma_{\bar{x}} \leq \bar{x} \leq \mu + 2.50\sigma_{\bar{x}}) = P(-2.50 \leq z \leq 2.50) = P(z \leq 2.50) - P(z \leq -2.50) = .9938 - .0062$

$= .9876$ or 98.76%

7.41 $P(\mu - 1.50\sigma_{\bar{x}} \le \bar{x} \le \mu + 1.50\sigma_{\bar{x}}) = P(-1.50 \le z \le 1.50) = P(z \le 1.50) - P(z \le -1.50) = .9332 - .0668$ $= .8664$ or 86.64%.

7.42 $\mu = 124$, $\sigma = 18$, $N = 10{,}000$, and $n = 36$

$\mu_{\bar{x}} = \mu = 124$

Since $n/N = 124/10{,}000 = .0124 < .05$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{36} = 3$

a. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (128.60 - 124)/3 = 1.53$

b. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (119.30 - 124)/3 = -1.57$

c. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (116.88 - 124)/3 = -2.37$

d. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (132.05 - 124)/3 = 2.68$

7.43 $\mu = 66$, $\sigma = 7$, $N = 205{,}000$, and $n = 49$

$\mu_{\bar{x}} = \mu = 66$

Since $n/N = 49/205{,}000 = .0002 < .05$, $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 7/\sqrt{49} = 1$

a. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (68.44 - 66)/1 = 2.44$

b. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (58.75 - 66)/1 = -7.25$

c. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (62.35 - 66)/1 = -3.65$

d. $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (71.82 - 66)/1 = 5.82$

7.44 $\mu = 75$, $\sigma = 14$, and $n = 20$

$\mu_{\bar{x}} = \mu = 75$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 14/\sqrt{20} = 3.13049517$

a. For $\bar{x} = 68.5$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (68.5 - 75)/3.13049517 = -2.08$

For $\bar{x} = 77.3$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (77.3 - 75)/3.13049517 = .73$

$P(68.5 < \bar{x} < 77.3) = P(-2.08 < z < .73) = P(z < .73) - P(z < -2.08) = .7673 - .0188 = .7485$

b. For $\bar{x} = 72.4$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (72.4 - 75)/3.13049517 = -.83$

$P(\bar{x} < 72.4) = P(z < -.83) = .2033$

7.45 $\mu = 48$, $\sigma = 8$, and $n = 16$

$\mu_{\bar{x}} = \mu = 48$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 8/\sqrt{16} = 2$

a. For $\bar{x} = 49.6$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (49.6 - 48)/2 = .80$

For $\bar{x} = 52.2$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (52.2 - 48)/2 = 2.10$

$P(49.6 < \bar{x} < 52.2) = P(.80 < z < 2.10) = P(z < 2.10) - P(z < .80) = .9821 - .7881 = .1940$

b. For $\bar{x} = 45.7$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (45.7 - 48)/2 = -1.15$

$P(\bar{x} > 45.7) = P(z > -1.15) = 1 - P(z \le -1.15) = 1 - .1251 = .8749$

7.46 $\mu = 60$, $\sigma = 10$, and $n = 40$

$\mu_{\bar{x}} = \mu = 60$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{40} = 1.58113883$

a. For $\bar{x} = 62.20$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (62.20 - 60)/1.58113883 = 1.39$

$P(\bar{x} < 62.20) = P(z < 1.39) = .9177$

b. For $\bar{x} = 61.4$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (61.4 - 60)/1.58113883 = .89$

For $\bar{x} = 64.2$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (64.2 - 60)/1.58113883 = 2.66$

$P(61.4 < \bar{x} < 64.2) = P(.89 < z < 2.66) = P(z < 2.66) - P(z < .89) = .9961 - .8133 = .1828$

7.47 $\mu = 90$, $\sigma = 18$, and $n = 64$

$\mu_{\bar{x}} = \mu = 90$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{64} = 2.25$

a. For $\bar{x} = 82.3$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (82.3 - 90)/2.25 = -3.42$

$P(\bar{x} < 82.3) = P(z < -3.42) = .0003$

b. For $\bar{x} = 86.7$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (86.7 - 90)/2.25 = -1.47$

$P(\bar{x} > 86.7) = P(z > -1.47) = 1 - P(z \le -1.47) = 1 - .0708 = .9292$

7.48 $\mu = 91.4$ grams, $\sigma = 93.25$ grams, n = 75

$\mu_{\bar{x}} = \mu = 91.4$ grams and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 93.25/\sqrt{75} = 10.76758252$ grams

a. For $\bar{x} = 80$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (80 - 91.4)/10.76758252 = -1.06$

$P(\bar{x} < 80) = P(z < -1.06) = .1446$

b. For $\bar{x} = 100$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (100 - 91.4)/10.76758252 = .80$

$P(\bar{x} > 100) = P(z > .80) = 1 - P(z \le .80) = 1 - .7881 = .2119$

c. For $\bar{x} = 95$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (95 - 91.4)/10.76758252 = .33$

For $\bar{x} = 102$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (102 - 91.4)/10.76758252 = .98$

$P(95 \le \bar{x} \le 102) = P(.33 \le z \le .98) = P(z \le .98) - P(z \le .33) = .8365 - .6293 = .2072$

7.49 $\mu = 3.02$, $\sigma = .29$, and $n = 20$

$\mu_{\bar{x}} = \mu = 3.02$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .29/\sqrt{20} = .06484597$

a. For $\bar{x} = 3.10$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (3.10 - 3.02)/.06484597 = 1.23$

$P(\bar{x} \ge 3.10) = P(z \ge 1.23) = 1 - P(z \le 1.23) = 1 - .8907 = .1093$

b. For $\bar{x} = 2.90$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (2.90 - 3.02)/.06484597 = -1.85$

$P(\bar{x} \le 2.90) = P(z \le -1.85) = .0322$

c. For $\bar{x} = 2.95$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (2.95 - 3.02)/.06484597 = -1.08$

For $\bar{x} = 3.11$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (3.11 - 3.02)/.06484597 = 1.39$

$P(2.95 \le \bar{x} \le 3.11) = P(-1.08 \le z \le 1.39) = P(z \le 1.39) - P(z \le -1.08) = .9177 - .1401 = .7776$

7.50 μ = 7.7 minutes, σ = 2.1 minutes, and n = 16

$\mu_{\bar{x}} = \mu = 7.7$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.1/\sqrt{16} = .525$ minutes

a. For $\bar{x} = 7$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (7 - 7.7)/.525 = -1.33$

For $\bar{x} = 8$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (8 - 7.7)/.525 = .57$

$P(7 < \bar{x} < 8) = P(-1.33 < z < .57) = P(z < .57) - P(z < -1.33) = .7157 - .0918 = .6239$

b. $P(\bar{x}$ within 1 minute of $\mu) = P(6.7 \le \bar{x} \le 8.7)$

For $\bar{x} = 6.7$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (6.7 - 7.7)/.525 = -1.90$

For $\bar{x} = 8.7$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (8.7 - 7.7)/.525 = 1.90$

$P(6.7 \le \bar{x} \le 8.7) = P(-1.90 \le z \le 1.90) = P(z \le 1.90) - P(z \le -1.90) = .9713 - .0287 = .9426$

c. $P(\bar{x}$ lower than μ by 1 minute or more$) = P(\bar{x} \le 6.7)$

For $\bar{x} = 6.7$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (6.7 - 7.7)/.525 = -.1.90$

$P(\bar{x} \le 6.7) = P(z \le -1.90) = .0287$

7.51 μ = \$55, σ = \$13.25, and n = 33

$\mu_{\bar{x}} = \mu = \$55$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 13.25/\sqrt{33} = \$2.30652894$

a. For $\bar{x} = 60$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (60 - 55)/2.30652894 = 2.17$

$P(\bar{x} > 60) = P(z \ge 2.17) = 1 - P(z \le 2.17) = 1 - .9850 = .0150$

b. For $\bar{x} = 52$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (52 - 55)/2.30652894 = -1.30$

$P(\bar{x} < 52) = P(z < -1.30) = .0968$

c. For $\bar{x} = 54$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (54 - 55)/2.30652894 = -.43$

For $\bar{x} = 57.99$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (57.99 - 55)/2.30652894 = 1.30$

$P(54 \le \bar{x} \le 57.99) = P(-.43 \le z \le 1.30) = P(z \le 1.30) - P(z \le -.43) = .9032 - .3336 = .5696$

7.52 μ = 8.4 hours, σ = 2.7 hours, and n = 45

$\mu_{\bar{x}} = \mu = 8.4$ hours and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.7/\sqrt{45} = .40249224$ hour

a. For $\bar{x} = 8$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (8 - 8.4)/.40249224 = -.99$

For $\bar{x} = 9$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (9 - 8.4)/.40249224 = 1.49$

$P(8 < \bar{x} < 9) = P(-.99 < z < 1.49) = P(z < 1.49) - P(z < -.99) = .9319 - .1611 = .7708$

b. For $\bar{x} = 8$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (8 - 8.4)/.40249224 = -.99$

$P(\bar{x} < 8) = P(z < -.99) = .1611$

7.53 μ = \$2840, σ = \$672, and n = 36

$\mu_{\bar{x}} = \mu = \$2840$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 672/\sqrt{36} = 112$

a. For $\bar{x} = 2600$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (2600 - 2840)/112 = -2.14$

For $\bar{x} = 2950$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (2950 − 2840)/112 = .98

$P(2600 < \bar{x} < 2950) = P(-2.14 < z < .98) = P(z < .98) - P(z < -2.14) = .8365 - .0162 = .8203$

b. For $\bar{x} = 3060$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (3060 − 2840)/112 = 1.96

$P(\bar{x} < 3060) = P(z < 1.96) = .9750$

7.54 $\mu = 200$ pieces, $\sigma = 145$ pieces, and $n = 84$

$\mu_{\bar{x}} = \mu = 200$ pieces and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 145/\sqrt{84} = 15.82079704$ pieces

a. For $\bar{x} = 160$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (160 − 200)/15.82079704 = −2.53

For $\bar{x} = 170$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (170 − 200)/15.82079704 = −1.90

$P(160 < \bar{x} < 170) = P(-2.53 < z < -1.90) = P(z < -1.90) - P(z < -2.53) = .0287 - .0057 = .0230$

b. For $\bar{x} = 120$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (120 − 200)/15.82079704 = −5.06

$P(\bar{x} > 120) = P(z \geq -5.06) = 1 - P(z \leq -5.06) = 1 - .0000 = 1$

c. For $\bar{x} = 150$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (150 − 200)/15.82079704 = −3.16

$P(\bar{x} \leq 150) = P(z \leq -3.16) = .0008$

7.55 $\mu = \$140$, $\sigma = \$30$, and $n = 75$

$\mu_{\bar{x}} = \mu = \$140$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 30/\sqrt{75} = \$3.46410162$

a. For $\bar{x} = 132$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (132 − 140)/3.46410162 = −2.31

For $\bar{x} = 136$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (136 − 140)/3.46410162 = −1.15

$P(132 < \bar{x} < 136) = P(-2.31 < z < -1.15) = P(z < -1.15) - P(z < -2.31) = .1251 - .0104 = .1147$

b. $P(\bar{x}$ within \$6 of $\mu) = P(134 \leq \bar{x} \leq 146)$

For $\bar{x} = 134$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (134 − 140)/3.46410162 = −1.73

For $\bar{x} = 146$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (146 − 140)/3.46410162 = 1.73

$P(134 \leq \bar{x} \leq 146) = P(-1.73 \leq z \leq 1.73) = P(z \leq 1.73) - P(z \leq -1.73) = .9582 - .0418 = .9164$

c. $P(\bar{x}$ greater than μ by at least \$4) = $P(\bar{x} \geq 144)$

For $\bar{x} = 144$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (144 − 140)/3.46410162 = 1.15

$P(\bar{x} \geq 144) = P(z \geq 1.15) = 1 - P(z \leq 1.15) = 1 - .8749 = .1251$

7.56 $\mu = \$3.084$, $\sigma = \$.263$, and $n = 47$

$\mu_{\bar{x}} = \mu = \$3.084$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .263/\sqrt{47} = \$.03836249$

a. For $\bar{x} = 3.00$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (3.00 – 3.084)/.03836249 = −2.19

$P(\bar{x} < 3.00) = P(z < -2.19) = .0143$

b. For $\bar{x} = 3.20$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (3.20 – 3.084)/.03836249 = 3.02

$P(\bar{x} > 3.20) = P(z > 3.02) = 1 - P(z \leq 3.02) = 1 - .9987 = .0013$

c. For $\bar{x} = 3.10$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (3.10 – 3.084)/.03836249 = .42

For $\bar{x} = 3.15$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (3.15 – 3.084)/.03836249 = 1.72

$P(3.10 \le \bar{x} \le 3.15) = P(.42 \le z \le 1.72) = P(z \le 1.72) - P(z \le .42) = .9573 - .6628 = .2945$

7.57 μ = 20.20 hours, σ = 2.60 hours, and n = 18

$\mu_{\bar{x}} = \mu = 20.20$ hours and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.60/\sqrt{18} = .61282588$ hour

a. $P(\bar{x}$ is not within one hour of $\mu) = P(\bar{x} < 19.20) + P(\bar{x} > 21.20) = 1 - P(19.20 \le z \le 21.20)$

For $\bar{x} = 19.20$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (19.20 – 20.20)/.61282588 = −1.63

For $\bar{x} = 21.20$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (21.20 – 20.20)/.61282588 = 1.63

$P(\bar{x} < 19.20) + P(\bar{x} > 21.20) = 1 - P(19.20 \le \bar{x} \le 21.20) = 1 - P(-1.63 \le z \le 1.63)$

$= 1 - [P(z \le 1.63) - P(z \le -1.63)] = 1 - [.9484 - .0516] = 1 - .8968 = .1032$

b. For $\bar{x} = 20.0$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (20.0 – 20.20)/.61282588 = −.33

For $\bar{x} = 20.5$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (20.5 – 20.20)/.61282588 = .49

$P(20.0 < \bar{x} < 20.5) = P(-.33 < z < .49) = P(z < .49) - P(z < -.33) = .6879 - .3707 = .3172$

c. For $\bar{x} = 22$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (22 – 20.20)/.61282588 = 2.94

$P(\bar{x} \ge 22) = P(z \ge 2.94) = 1 - P(z < 2.94) = 1 - .9984 = .0016$

d. For $\bar{x} = 21$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (21 – 20.20)/.61282588 = 1.31

$P(\bar{x} \le 21) = P(z \le 1.31) = .9049$

7.58 $\bar{x}$ = 2250 hours, σ = 150 hours, and n = 100

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 150/\sqrt{100} = 15$ hours

We are to find $P(\mu - 25 \le \bar{x} \le \mu + 25)$

For $\bar{x} = \mu - 25$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (\mu - 25 - \mu)/15 = -1.67$

For $\bar{x} = \mu + 25$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (\mu + 25 - \mu)/15 = 1.67$

$P(\mu - 25 \le \bar{x} \le \mu + 25) = P(-1.67 \le z \le 1.67) = P(z \le 1.67) - P(z \le -1.67) = .9525 - .0475 = .9050$

7.59 μ = 3 inches, σ = .1 inch, and n = 25

$\mu_{\bar{x}} = \mu = 3$ inches and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .1/\sqrt{25} = .02$ inch

For $\bar{x} = 2.95$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (2.95 − 3)/.02 = −2.50

For $\bar{x} = 3.05$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (3.05 − 3)/.02 = 2.50

$P(x < 2.95) + P(x > 3.05) = 1 - [P(2.95 \le x \le 3.05)] = 1 - [P(-2.50 \le z \le 2.50)]$

$= 1 - [P(z \le 2.50) - P(z \le -2.50)] = 1 - [.9938 - .0062] = 1 - .9876 = .0124$

Sections 7.6 - 7.7

7.60 $p = 640/1000 = .64$ and $\hat{p} = 24/40 = .60$

7.61 $p = 600/5000 = .12$ and $\hat{p} = 18/120 = .15$

7.62 Number with characteristic in population = (18,700)(.3) = 5610

Number with characteristic in sample = (250)(.25) = 62.5

7.63 Number with characteristic in population = (9500)(.75) = 7125

Number with characteristic in sample = (400)(.78) = 312

7.64 a. $\mu_{\hat{p}} = p$

b. $\sigma_{\hat{p}} = \sqrt{pq/n}$

c. The sampling distribution of $\hat{p}$ is approximately normal if $np > 5$ and $nq > 5$.

7.65 Sampling error = $\hat{p} - p = .66 - .71 = -.05$

7.66 Sampling error = $\hat{p} - p = .33 - .29 = .04$

7.67 The estimator of *p* is the sample proportion $\hat{p}$.

The sample proportion $\hat{p}$ is an unbiased estimator of *p*, since the mean of $\hat{p}$ is equal to *p*.

7.68 The sample proportion $\hat{p}$ is a consistent estimator of *p*, since $\sigma_{\hat{p}}$ decreases as the sample size increases.

7.69 $\sigma_{\hat{p}} = \sqrt{pq/n}$, hence $\sigma_{\hat{p}}$ decreases as *n* increases.

7.70 $p = .63$ and $q = 1 - p = 1 - .63 = .37$

a. $n = 100$, $\mu_{\hat{p}} = p = .63$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.63)(.37)/100} = .048$

b. $n = 900$, $\mu_{\hat{p}} = p = .63$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.63)(.37)/900} = .016$

7.71 $p = .21$ and $q = 1 - p = 1 - .21 = .79$

a. $n = 400$, $\mu_{\hat{p}} = p = .21$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.21)(.79)/400} = .020$

b. $n = 750$, $\mu_{\hat{p}} = p = .21$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.21)(.79)/750} = .015$

7.72 $p = .12$, $q = 1 - p = 1 - .12 = .88$, and $N = 4000$

a. $n/N = 800/4000 = .20 > .05$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{(.12)(.88)}{800}}\sqrt{\frac{4000-800}{4000-1}} = .010$$

b. Since $n/N = 30/4000 = .0075 < .05$, $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.12)(.88)/30} = .059$

7.73 $p = .47$, $q = 1 - p = 1 - .47 = .53$, and $N = 1400$

a. $n/N = 90/1400 = .064 > .05$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}\sqrt{\frac{N-n}{N-1}} = \sqrt{\frac{(.47)(.53)}{90}}\sqrt{\frac{1400-90}{1400-1}} = .051$$

b. Since $n/N = 50/1400 = .036 < .05$, $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.47)(.53)/50} = .071$

7.74 A sample is considered large enough to apply the central limit theorem if $np > 5$ and $nq > 5$.

7.75 a. $np = (400)(.28) = 112$ and $nq = (400)(.72) = 288$

Since $np > 5$ and $nq > 5$, the central limit theorem applies.

b. $np = (80)(.05) = 4$; since $np < 5$, the central limit theorem does not apply.

c. $np = (60)(.12) = 7.2$ and $nq = (60)(.88) = 52.8$

Since $np > 5$ and $nq > 5$, the central limit theorem applies.

d. $np = (100)(.035) = 3.5$; since $np < 5$, the central limit theorem does not apply.

7.76 a. $np = (20)(.45) = 9$ and $nq = (20)(.55) = 11$

Since $np > 5$ and $nq > 5$, the central limit theorem applies.

b. $np = (75)(.22) = 16.5$ and $nq = (75)(.78) = 58.5$

Since $np > 5$ and $nq > 5$, the central limit theorem applies.

c. $np = (350)(.01) = 3.5$; since $np < 5$, the central limit theorem does not apply.

d. $np = (200)(.022) = 4.4$; since $np < 5$, the central limit theorem does not apply.

7.77 a. $p = 4/6 = .667$

b. ${}_6C_5 = 6$.

c & d. Let: G = good TV set and D = defective TV set

Let the six TV sets be denoted as: $1 = G$, $2 = G$, $3 = D$, $4 = D$, $5 = G$, and $6 = G$. The six possible samples, their sample proportions, and the sampling errors are given in the table below.

Sample	TV sets	$\hat{p}$	Sampling error
1, 2, 3, 4, 5	G, G, D, D, G	3/5=.60	.60 – .667 = –.067
1, 2, 3, 4, 6	G, G, D, D, G	3/5=.60	.60 – .667 = –.067
1, 2, 3, 5, 6	G, G, D, G, G	4/5=.80	.80 – .667 = .133
1, 2, 4, 5, 6	G, G, D, G, G	4/5=.80	.80 – .667 = .133
1, 3, 4, 5, 6	G, D, D, G, G	3/5=.60	.60 – .667 = –.067
2, 3, 4, 5, 6	G, D, D, G, G	3/5=.60	.60 – .667 = –.067

$\hat{p}$	f	Relative Frequency
.60	4	4/6=.667
.80	2	2/6=.333
	$\sum f = 6$	

$\hat{p}$	$P(\hat{p})$
.60	.667
.80	.333

7.78 a. $p = 2/5 = .40$

b. ${}_5C_3 = 10$.

c & d. Let the five fires of the season be denoted as:

A = arson, B = accident, C = accident, D = arson, and E = accident

The following table lists all the possible samples of size 3, the sample proportions, and the sampling errors.

Sample	Cause	$\hat{p}$	Sampling error
A, B, C	arson, accident, accident	1/3 = .333	.333 – .40 = –.067
A, B, D	arson, accident, arson	2/3 = .667	.667 – .40 = .267
A, B, E	arson, accident, accident	1/3 = .333	.333 – .40 = –.067
A, C, D	arson, accident, arson	2/3 = .667	.667 – .40 = .267
A, C, E	arson, accident, accident	1/3 = .333	.333 – .40 = –.067
A, D, E	arson, arson, accident	2/3 = .667	.667 – .40 = .267
B, C, D	accident, accident, arson	1/3 = .333	.333 – .40 = –.067
B, C, E	accident, accident, accident	0/3 = .000	.000 – .40 = –.400
B, D, E	accident, arson, accident	1/3 = .333	.333 – .40 = –.067
C, D, E	accident, arson, accident	1/3 = .333	.333 – .40 = –.067

$\hat{p}$	f	Relative Frequency
.000	1	1/10 = .10
.333	6	6/10 = .60
.667	3	3/10 = .30

$\hat{p}$	$P(\hat{p})$
.000	.10
.333	.60
.667	.30

7.79 $p = .30$, $q = 1 - p = 1 - .30 = .70$, and $n = 180$

$\mu_{\hat{p}} = p = .30$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.30)(.70)/180} = .0342$

$np = (180)(.30) = 54$ and $nq = (180)(.70) = 126$

Since $np > 5$ and $nq > 5$, the sampling distribution of $\hat{p}$ is approximately normal.

7.80 $p = .43$, $q = 1 - p = 1 - .43 = .57$, and $n = 110$

$\mu_{\hat{p}} = p = .43$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.43)(.57)/110} = .0472$

$np = (110)(.47) = 47.3$ and $nq = (110)(.57) = 62.7$

Since $np > 5$ and $nq > 5$, the sampling distribution of $\hat{p}$ is approximately normal.

7.81 $p = .561$, $q = 1 - p = 1 - .561 = .439$, and $n = 340$

$\mu_{\hat{p}} = p = .561$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.561)(.439)/340} = .0269$

$np = (340)(.561) = 190.74$ and $nq = (340)(.439) = 149.26$

Since $np > 5$ and $nq > 5$, the sampling distribution of $\hat{p}$ is approximately normal.

7.82 $p = .231$, $q = 1 - p = 1 - .231 = .769$, and $n = 460$

$\mu_{\hat{p}} = p = .231$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.231)(.769)/460} = .0197$

$np = (460)(.231) = 106.26$ and $nq = (460)(.769) = 353.74$

Since $np > 5$ and $nq > 5$, the sampling distribution of $\hat{p}$ is approximately normal.

Section 7.8

7.83 $P(p - 2.0\sigma_{\hat{p}} \le \hat{p} \le p + 2.0\sigma_{\hat{p}}) = P(-2.00 \le z \le 2.00) = P(z \le 2.00) - P(z \le -2.00) = .9772 - .0228 =$ .9544 or 95.44%.

7.84 $P(p - 3.0\sigma_{\hat{p}} \le \hat{p} \le p + 3.0\sigma_{\hat{p}}) = P(-3.00 \le z \le 3.00) = P(z \le 3.00) - P(z \le -3.00) = .9987 - .0013 =$ .9974 or 99.74%.

7.85 $p = .59$, $q = 1 - p = 1 - .59 = .41$, $N = 30{,}000$, and $n = 400$

$n/N = 100/30{,}000 = .033 < .05$, $np = (100)(.59) = 59 > 5$, $nq = (100)(41) = 41 > 5$

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.59)(.41)/100} = .04918333$

a. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.56 - .59)/.04918333 = -.61$

b. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.68 - .59)/.04918333 = 1.83$

c. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.53 - .59)/.04918333 = -1.22$

d. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.65 - .59)/.04918333 = 1.22$

7.86 $p = .25$, $q = 1 - p = 1 - .25 = .75$, $N = 18{,}000$, and $n = 70$

$n/N = 70/18{,}000 = .004 < .05$, $np = (70)(.25) = 17.5 > 5$, $nq = (70)(.75) = 52.5 > 5$

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.25)(.75)/70} = .05175492$

a. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.26 - .25)/.05175492 = .19$

b. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.32 - .25)/.05175492 = 1.35$

c. $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.17 - .25)/.05175492 = -1.55$

d. $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.20 − .25)/.05175492 = −.97

7.87 p = .30, $q = 1 - p$ = 1 − .30 = .70, and n = 180

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.30)(.70)/180}$ = .03415650

a. For $\hat{p}$ = .35: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.35 − .30)/.03415650 = 1.46

$P(\hat{p} > .35) = P(z > 1.46) = 1 - P(z \le 1.46)$ = 1 − .9279 = .0721

b. For $\hat{p}$ = .22: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.22 − .30)/.03415650 = −2.34

For $\hat{p}$ = .27: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.27 − .30)/.03415650 = −.88

$P(.22 < \hat{p} < .27) = P(-2.34 < z < -.88) = P(z < -.88) - P(z < -2.34)$ = .1894 − .0096 = .1798

7.88 p = .64, $q = 1 - p$ = 1 − .64 = .36, and n = 50

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.64)(.36)/50}$ = .06788225

a. For $\hat{p}$ = .54: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.54 − .64)/.06788225 = −1.47

For $\hat{p}$ = .61: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.61 − .64)/.06788225 = −.44

$P(.54 < \hat{p} < .61) = P(-1.47 < z < -.44) = P(z < -.44) - P(z < -1.47)$ = .3300 − .0708 = .2592

b. For $\hat{p}$ = .71: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.71 − .64)/.06788225 = 1.03

$P(\hat{p} > .71) = P(z > 1.03) = 1 - P(z \le 1.03)$ = 1 − .8485 = .1515

7.89 p = .43, $q = 1 - p$ = 1 − .43 = .57, and n = 110

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.43)(.57)/110}$ = .04720362

a. For $\hat{p}$ = .30: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.30 − .43)/.04720362 = −2.75

$P(\hat{p} < .30) = P(z < -2.75)$ = .0030

b. For $\hat{p}$ = .45: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.45 − .43)/.04720362 = .42

For $\hat{p}$ = .50: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.50 − .43)/.04720362 = 1.48

$P(.45 < \hat{p} < .50) = P(.42 < z < 1.48) = P(z < 1.48) - P(z < .42)$ = .9306 − .6628 = .2678

7.90 p = .85, $q = 1 - p$ = 1 − .85 = .15, and n = 100

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.85)(.15)/100}$ = .03570714

a. For $\hat{p}$ = .81: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.81 − .85)/.03570714 = −1.12

For $\hat{p}$ = .88: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.88 − .85)/.03570714 = .84

$P(.81 < \hat{p} < .88) = P(-1.12 < z < .84) = P(z < .84) - P(z < -1.12)$ = .7995 − .1314 = .6681

b. For $\hat{p} = .87$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.87 − .85)/.03570714 = .56

$P(\hat{p} < .87) = P(z < .56) = .7123$

7.91 $p = .06$, $q = 1 - p = 1 - .06 = .94$, and $n = 100$

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.06)(.94)/100} = .02374868$

For $\hat{p} = .08$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.08 − .06)/.02374868 = .84

$P(\hat{p} \geq .08) = P(z \geq .84) = 1 - P(z \leq .84) = 1 - .7995 = .2005$

7.92 $p = .80$, $q = 1 - p = 1 - .80 = .20$, and $n = 100$

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.80)(.20)/100} = .04$

a. $P(\hat{p}$ within .05 of $p) = P(.75 \leq \hat{p} \leq .85)$

For $\hat{p} = .75$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.75 − .80)/.04 = −1.25

For $\hat{p} = .85$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.85 − .80)/.04 = 1.25

$P(.75 \leq \hat{p} \leq .85) = P(-1.25 \leq z \leq 1.25) = P(z \leq 1.25) - P(z \leq -1.25) = .8944 - .1056 = .7888$

b. $P(\hat{p}$ is less than p by .06 or more) $= P(\hat{p} \leq .74)$

For $\hat{p} = .74$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.74 − .80)/.04 = −1.50

$P(\hat{p} \leq .74) = P(z \leq -1.50) = .0668$

c. $P(\hat{p}$ greater than p by .07 or more) $= P(\hat{p} \geq .87)$

For $\hat{p} = .87$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.87 − .80)/.04 = 1.75

$P(\hat{p} \geq .87) = P(z \geq 1.75) = 1 - P(z \leq 1.75) = 1 - .9599 = .0401$

Supplementary Exercises

7.93 $\mu = 750$ hours, $\sigma = 55$ hours, and $n = 25$

$\mu_{\bar{x}} = \mu = 750$ hours and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 55/\sqrt{25} = 11$ hours

The sampling distribution of $\bar{x}$ is normal because the population is normally distributed.

7.94 $\mu = 225$ minutes, $\sigma = 62$ minutes, and $n = 20$

$\mu_{\bar{x}} = \mu = 225$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 62/\sqrt{20} = 13.864$ minutes

The sampling distribution of $\bar{x}$ is normal because the population is normally distributed.

7.95 $\mu = 750$ hours, $\sigma = 55$ hours, and $n = 25$

$\mu_{\bar{x}} = \mu = 750$ hours and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 55/\sqrt{25} = 11$ hours

a. For $\bar{x} = 735$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (735 − 750)/11 = −1.36

$P(\bar{x} > 735) = P(z > -1.36) = 1 - P(z \le -1.36) = 1 - .0869 = .9131$

b. For $\bar{x} = 725$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (725 − 750)/11 = −2.27

For $\bar{x} = 740$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (740 − 750)/11 = −.91

$P(725 < \bar{x} < 740) = P(-2.27 < z < -.91) = P(z < -.91) - P(z < -2.27) = .1814 - .0116 = .1698$

c. $P(\bar{x}$ within 15 hours of $\mu) = P(735 \le \bar{x} \le 765)$

For $\bar{x} = 735$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (735 − 750)/11 = −1.36

For $\bar{x} = 765$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (765 − 750)/11 = 1.36

$P(735 \le \bar{x} \le 765) = P(-1.36 \le z \le 1.36) = P(z \le 1.36) - P(z \le -1.36) = .9131 - .0869 = .8262$

d. $P(\bar{x}$ is less than μ by 20 hours or more) = $P(\bar{x} < 730)$

For $\bar{x} = 730$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (730 − 750)/11 = −1.82

$P(\bar{x} < 730) = P(z < -1.82) = .0344$

7.96 $\mu = 225$ minutes, $\sigma = 62$ minutes, and $n = 20$

$\mu_{\bar{x}} = \mu = 225$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 62/\sqrt{20} = 13.86362146$ minutes

a. For $\bar{x} = 200$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (200 − 225)/13.86362146 = −1.80

$P(\bar{x} < 200) = P(z < -1.80) = .0359$

b. For $\bar{x} = 230$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (230 − 225)/13.86362146 = .36

For $\bar{x} = 240$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (240 − 225)/13.86362146 = 1.08

$P(230 < \bar{x} < 240) = P(.36 < z < 1.08) = P(z < 1.08) - P(z < .36) = .8599 - .6406 = .2193$

c. $P(\bar{x}$ within 20 minutes of $\mu) = P(205 \le \bar{x} \le 245)$

For $\bar{x} = 205$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (205 − 225)/13.86362146 = −1.44

For $\bar{x} = 245$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (245 − 225)/13.86362146 = 1.44

$P(205 \le \bar{x} \le 245) = P(-1.44 \le z \le 1.44) = P(z \le 1.44) - P(z \le -1.44) = .9251 - .0749 = .8502$

d. For $\bar{x} = 260$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (260 − 225)/13.86362146 = 2.52

$P(\bar{x} > 260) = P(z > 2.52) = 1 - P(z \le 2.52) = 1 - .9941 = .0059$

7.97 $\mu = 190$ minutes, $\sigma = 53.4$ minutes, and $n = 12$

$\mu_{\bar{x}} = \mu = 190$ minutes and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 53.4/\sqrt{12} = 15.41525219$ minutes

a. $P(\bar{x}$ within 10 minutes of $\mu) = P(180 \le \bar{x} \le 200)$

For $\bar{x} = 180$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (180 − 190)/15.41525219 = −.65

For $\bar{x} = 200$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (200 − 190)/15.41525219 = .65

$P(180 \le \bar{x} \le 200) = P(-.65 \le z \le .65) = P(z \le .65) - P(z \le -.65) = .7422 - .2578 = .4844$

b. For $\bar{x} = 240$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (240 − 190)/15.41525219 = 3.24

$P(\bar{x} > 240) = P(z > 3.24) = 1 - P(z \leq 3.24) = 1 - .9994 = .0006$

c. $P(\bar{x}$ at least 20 minutes different than $\mu) = P(\bar{x} < 170) + P(\bar{x} > 210)$

For $\bar{x} = 170$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (170 – 190)/15.41525219 = −1.30

For $\bar{x} = 210$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (210 – 190)/15.41525219 = 1.30

$P(\bar{x} < 170) + P(\bar{x} > 210) = 1 - P(170 \leq \bar{x} \leq 210) = 1 - P(-1.30 \leq z \leq 1.30)$

$= 1 - [P(z \leq 1.30) - P(z \leq -1.30)] = 1 - [.9032 - .0968] = 1 - .8064 = .1936$

d. For $\bar{x} = 207$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (207 − 190)/15.41525219 = 1.10

$P(\bar{x} < 207) = P(z < 1.10) = .8643$

7.98 $\mu = 64$ ounces, $\sigma = .4$ ounce, and $n = 16$

$\mu_{\bar{x}} = \mu = 64$ ounces and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .4/\sqrt{16} = .1$ ounce

For $\bar{x} = 63.75$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (63.75 − 64)/.1 = −2.50

For $\bar{x} = 64.25$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}}$ = (64.25 − 64)/.1 = 2.50

$P(\bar{x} < 63.75) + P(\bar{x} > 64.25) = 1 - [P(63.75 \leq \bar{x} \leq 64.25)] = 1 - [P(-2.50 \leq z \leq 2.50)]$

$= 1 - [P(z \leq 2.50) - P(z \leq -2.50)] = 1 - [.9938 - .0062] = 1 - .9876 = .0124$

7.99 $p = .88$, $q = 1 - p = 1 - .88 = .12$, and $n = 80$

$\mu_{\hat{p}} = p = .88$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.88)(.12)/80} = .03633180$

$np = (80)(.88) = 70.4 > 5$, $nq = (80)(.12) = 9.6 > 5$

Since np and nq are both greater than 5, the sampling distribution of $\hat{p}$ is approximately normal.

7.100 $p = .70$, $q = 1 - p = 1 - .70 = .30$, and $n = 400$

$\mu_{\hat{p}} = p = .70$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.70)(.30)/400} = .02291288$

$np = (400)(.70) = 280 > 5$, $nq = (400)(.30) = 120 > 5$

Since np and nq are both greater than 5, the sampling distribution of $\hat{p}$ is approximately normal.

7.101 $p = .70$, $q = 1 - p = 1 - .70 = .30$, and $n = 400$

$\mu_{\hat{p}} = p = .70$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.70)(.30)/400} = .02291288$

a. i. For $\hat{p} = .65$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.65 − .70)/.02291288 = −2.18

$P(\hat{p} < .65) = P(z < -2.18) = .0146$

ii. For $\hat{p} = .73$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.73 − .70)/.02291288 = 1.31

For $\hat{p} = .76$: $z = (\hat{p} - p)/\sigma_{\hat{p}}$ = (.76 − .70)/.02291288 = 2.62

$P(.73 < \hat{p} < .76) = P(1.31 < z < 2.62) = P(z < 2.62) - P(z < 1.31) = .9956 - .9049 = .0907$

b. $P(\hat{p}$ within .06 of $p) = P(.64 \leq \hat{p} \leq .76)$

For $\hat{p} = .64$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.64 - .70)/.02291288 = -2.62$

For $\hat{p} = .76$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.76 - .70)/.02291288 = 2.62$

$P(.64 \leq \hat{p} \leq .76) = P(-2.62 \leq z \leq 2.62) = P(z \leq 2.62) - P(z \leq -2.62) = .9956 - .0044 = .9912$

c. $P(\hat{p}$ greater than p by .05 or more) $= P(\hat{p} \geq .75)$

For $\hat{p} = .75$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.75 - .70)/.02291288 = 2.18$

$P(\hat{p} \geq .75) = P(z \geq 2.18) = 1 - P(z \leq 2.18) = 1 - .9854 = .0146$

7.102 $p = .15$, $q = 1 - p = 1 - .15 = .85$, and $n = 800$

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.15)(.85)/800} = .01262438$

a. $P(\hat{p}$ within .02 of $p) = P(.13 \leq \hat{p} \leq .17)$

For $\hat{p} = .13$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.13 - .15)/.01262438 = -1.58$

For $\hat{p} = .17$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.17 - .15)/.01262438 = 1.58$

$P(.13 \leq \hat{p} \leq .17) = P(-1.58 \leq z \leq 1.58) = P(z \leq 1.58) - P(z \leq -1.58) = .9429 - .0571 = .8858$

b. $P(\hat{p}$ is not within .02 of $p) = 1 - P(\hat{p}$ is within .02 of $p) = 1 - .8858 = .1142$

c. $P(\hat{p}$ greater than p by .025 or more) $= P(\hat{p} \geq .175)$

For $\hat{p} = .175$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.175 - .15)/.01262438 = 1.98$

$P(\hat{p} \geq .175) = P(z \geq 1.98) = 1 - P(z \leq 1.98) = 1 - .9761 = .0239$

d. $P(\hat{p}$ is less than p by .03 or more) $= P(\hat{p} \leq .12)$

For $\hat{p} = .12$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.12 - .15)/.01262438 = -2.38$

$P(\hat{p} \leq .12) = P(z \leq -2.38) = .0087$

7.103 $\sigma = \$2,845,000$, and $n = 32$

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2.845{,}000/\sqrt{26} = \$557{,}950.40$

The required probability is: $P(\mu - 500{,}000 \leq \bar{x} \leq \mu + 500{,}000)$

For $\bar{x} = \mu - 500{,}000$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (\mu - 500{,}000 - \mu)/557{,}950.40 = -.90$

For $\bar{x} = \mu + 500{,}000$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (\mu + 500{,}000 - \mu)/557{,}950.40 = .90$

$P(\mu - 500{,}000 \leq \bar{x} \leq \mu + 500{,}000) = P(-.90 \leq z \leq .90) = P(z \leq .90) - P(z \leq -.90) = .8159 - .1841$
$= .6318$

7.104 Given $P(\bar{x} > 90) = .15$ and $P(\bar{x} < 65) = .30$, the corresponding z values are approximately $z = 1.04$ and $z = -.52$, respectively. First, we use $x = \mu + z\sigma$ to find σ. We have $90 = \mu + 1.04\sigma$ and

$65 = \mu + (-.52)\sigma$. Subtracting gives $25 = 1.56\sigma$, so $\sigma = 16.0256$. Since $65 = \mu + (-.52)\sigma$, we have $65 = \mu + (-.52)(16.0256)$ so $\mu = 65 + (.52)(16.0256) \approx 73.33$.

7.105 $\mu = c$ and $\sigma = .8$ ppm

We want $P(\mu - .5 \le \bar{x} \le \mu + .5) = .95$. The corresponding z value is 1.96; then $1.96\,\sigma_{\bar{x}} = .5$ and $\sigma_{\bar{x}} = .255$. Since $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, $n = (\sigma/\sigma_{\bar{x}})^2 = (.8/.255)^2 = 9.84$. Thus, 10 measurements are necessary.

7.106 a. $p = .60$, $q = 1 - p = 1 - .60 = .40$, and $n = 25$

Since $np = (25)(.60) = 15 > 5$, $nq = (25)(.40) = 10 > 5$, we can use the normal approximation to the binomial where $\mu = np = (25)(.60) = 15$ and $\sigma = \sqrt{npq} = \sqrt{(25)(.60)(.40)} = 2.44948974$.

For $x = 12.5$: $z = (x - \mu)/\sigma = (12.5 - 15)/2.44948974 = -1.02$

$P(x > 12.5) = P(z > -1.02) = 1 - P(z \le -1.02) = 1 - .1539 = .8461$

b. For .95 or higher, $z = -1.65$. Now,

$z = (\hat{p} - p)/\sigma_{\hat{p}}$, so $\sigma_{\hat{p}} = \dfrac{\hat{p} - p}{z} = \dfrac{.5 - .6}{-1.65} = .0606$. Then, since

$\sigma_{\hat{p}} = \sqrt{pq/n}$, $n = pq/(\sigma_{\hat{p}})^2 = .60(.40)/(.0606)^2 = 65.35$. The reporter should take a sample of at least 66 voters.

7.107 a. $p = .53$, $q = 1 - p = 1 - .53 = .47$, and $n = 200$

$\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.53)(.47)/200} = .03529164$

For $\hat{p} = .50$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.50 - .53)/.03529164 = -.85$

$P(\hat{p} > .50) = P(z > -.85) = 1 - P(z \le -.85) = 1 - .1977 = .8023$

b. For .95 or higher, $z = -1.65$. Now,

$z = (\hat{p} - p)/\sigma_{\hat{p}}$, so $\sigma_{\hat{p}} = \dfrac{\hat{p} - p}{z} = \dfrac{.5 - .53}{-1.65} = .01818182$. Then, since

$\sigma_{\hat{p}} = \sqrt{pq/n}$, $n = pq/(\sigma_{\hat{p}})^2 = .53(.47)/(.01818182)^2 = 753.53$. The politician should take a sample of at least 754 voters.

7.108 $\mu = 290$ feet, $\sigma = 10$ feet, and $n = 3$

$\mu_{\bar{x}} = \mu = 290$ feet and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 10/\sqrt{3} = 5.77350269$ feet

P (total length of three throws exceeds 885) = P(mean length of three throws exceeds 885/3)

$= P(\bar{x} > 295)$

For $\bar{x} = 295$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (295 - 290)/5.77350269 = .87$

$P(\bar{x} > 295) = P(z > .87) = 1 - P(z \le .87) = 1 - .8078 = .1922$

7.109 $\mu = 160$ pounds, $\sigma = 25$ pounds, and $n = 35$

$\mu_{\bar{x}} = \mu = 160$ pounds and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 25/\sqrt{35} = 4.22577127$ pounds

Since $n \geq 30$, $\bar{x}$ is approximately normally distributed.

P (sum of 35 weights exceeds 6000 pounds) = P (mean weight exceeds 6000/35) = $P(\bar{x} > 171.43)$

For $\bar{x} = 171.43$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (171.43 - 160)/4.22577127 = 2.70$

$P(\bar{x} > 171.43) = P(z > 2.70) = 1 - P(z \leq 2.70) = 1 - .9965 = .0035$

7.110 $p = .415$, and $q = 1 - p = 1 - .415 = .585$

a. $n = 70$, $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.415)(.585)/70} = .05889155$

For $\hat{p} = .514$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.514 - .415)/.05889155 = 1.68$

$P(\hat{p} \geq .514) = P(z \geq 1.68) = 1 - P(z \leq 1.68) = 1 - .9535 = .0465$

b. $n = 250$, $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.415)(.585)/250} = .03116248$

For $\hat{p} = .514$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.514 - .415)/.03116248 = 3.18$

$P(\hat{p} \geq .514) = P(z \geq 3.18) = 1 - P(z \leq 3.18) = 1 - .9993 = .0007$

c. For .01 in the right tail, $z = 2.33$. Now,

$z = (\hat{p} - p)/\sigma_{\hat{p}}$, so $\sigma_{\hat{p}} = \dfrac{\hat{p} - p}{z} = \dfrac{.514 - .415}{2.33} = .04248927$. Then, since

$\sigma_{\hat{p}} = \sqrt{pq/n}$, $n = pq/(\sigma_{\hat{p}})^2 = .415(.585)/(.04248927)^2 = 134.48$. Thus, the minimum sample size is 135.

7.111

Sample	Scores	Sample Median
ABC	70, 78, 80	78
ABD	70, 78, 80	78
ABE	70, 78, 95	78
ACD	70, 80, 80	80
ACE	70, 80, 95	80
ADE	70, 80, 95	80
BCD	78, 80, 80	80
BCE	78, 80, 95	80
BDE	78, 80, 95	80
CDE	80, 80, 95	80

Mean of the sample medians: (78 + 78 + 78 + 80 + 80 + 80 + 80 + 80 + 80 + 80)/10 = 79.4. This is not equal to the population mean of 80.6. We could change the 78 to 80 and change the 95 to 90. Then, each of the sample medians would be equal to 80, and therefore have a mean of 80. The population mean would become μ = (70 + 80 + 80 + 80 + 80 + 80 + 80 + 80 + 80 + 90)/10 = 80.

7.112 As n gets larger, the distribution becomes approximately normal and $\sigma_{\bar{x}}$ decreases. Then $(a-\mu)/\sigma_{\bar{x}}$ becomes smaller and $(b-\mu)/\sigma_{\bar{x}}$ becomes larger. Hence,

$P(a \le \bar{x} \le b) = P\left(\frac{a-\mu}{\sigma_{\bar{x}}} \le z \le \frac{b-\mu}{\sigma_{\bar{x}}}\right)$ increases.

Self – Review Test

1. b	**2.** b	**3.** a	**4.** a	**5.** b	**6.** b
7. c	**8.** a	**9.** a	**10.** a	**11.** c	**12.** a

13. According to the central limit theorem, for a large sample size, the sampling distribution of the sample mean is approximately normal irrespective of the shape of the population distribution. The mean and standard deviation of the sampling distribution of the sample mean are $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma/\sqrt{n}$, respectively. The sample size is usually considered to be large if $n \ge 30$. From the same theorem, the sampling distribution of $\hat{p}$ is approximately normal for sufficiently large samples. In the case of proportion, the sample is sufficiently large if $np > 5$ and $nq > 5$.

14. $\mu = 145$ pounds and $\sigma = 18$ pounds

a. $\mu_{\bar{x}} = \mu = 145$ pounds and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{25} = 3.60$ pounds

b. $\mu_{\bar{x}} = \mu = 145$ pounds and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 18/\sqrt{100} = 1.80$ pounds

In both cases the sampling distribution of $\bar{x}$ is approximately normal because the population has an approximate normal distribution.

15. $\mu = 45{,}000$ miles and $\sigma = 2360$ miles

a. $\mu_{\bar{x}} = \mu = 45{,}000$ miles and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2360/\sqrt{20} = 527.71$ miles

Since the population has an unknown distribution and $n < 30$, we can draw no conclusion about the shape of the sampling distribution of $\bar{x}$.

b. $\mu_{\bar{x}} = \mu = 45{,}000$ miles and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2360/\sqrt{65} = 292.72$ miles

Since $n \ge 30$, the sampling distribution of $\bar{x}$ is approximately normal.

16. $\mu = 45{,}000$ miles and $\sigma = 2360$ miles

$\mu_{\bar{x}} = \mu = 45{,}000$ miles and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = 2360/\sqrt{65} = 292.7219736$ miles

a. For $\bar{x} = 44{,}500$: $z = (\bar{x}-\mu)/\sigma_{\bar{x}} = (44{,}500 - 45{,}000)/292.7219736 = -1.71$

For $\bar{x} = 44{,}750$: $z = (\bar{x}-\mu)/\sigma_{\bar{x}} = (44{,}750 - 45{,}000)/292.7219736 = -.85$

$P(44{,}500 < \bar{x} < 44{,}750) = P(-1.71 < z < -.85) = P(z < -.85) - P(z < -1.71) = .1977 - .0436 = .1541$

b. $P(\bar{x}$ within 180 miles of $\mu) = P(44{,}820 \le \bar{x} \le 45{,}180)$

For $\bar{x} = 44{,}820$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (44{,}820 - 45{,}000)/292.7219736 = -.61$

For $\bar{x} = 45{,}180$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (45{,}180 - 45{,}000)/292.7219736 = .61$

$P(44{,}820 \le \bar{x} \le 45{,}180) = P(-.61 \le z \le .61) = P(z \le .61) - P(z \le -.61) = .7291 - .2709 = .4582$

c. For $\bar{x} = 46{,}000$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (46{,}000 - 45{,}000)/292.7219736 = 3.42$

$P(\bar{x} \ge 46{,}000) = P(z \ge 3.42) = 1 - P(z \le 3.42) = 1 - .9997 = .0003$

d. $P(\bar{x}$ not within 400 miles of $\mu) = P(\bar{x} \le 44{,}600) + P(\bar{x} \ge 45{,}400)$

For $\bar{x} = 44{,}600$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (44{,}600 - 45{,}000)/292.7219736 = -1.37$

For $\bar{x} = 45{,}400$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (45{,}400 - 45{,}000)/292.7219736 = 1.37$

$P(\bar{x} \le 44{,}600) + P(\bar{x} \ge 45{,}400) = 1 - P(44{,}600 < \bar{x} < 45{,}400) = 1 - P(-1.37 \le z \le 1.37)$

$= 1 - [P(z \le 1.37) - P(z \le -1.37)] = 1 - [.9147 - .0853] = 1 - .8294 = .1706$

e. For $\bar{x} = 44{,}300$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (44{,}300 - 45{,}000)/292.7219736 = -2.39$

$P(\bar{x} < 44{,}300) = P(z < -2.39) = .0084$

17. $\mu = 16$ ounces, $\sigma = .18$ ounce, and $n = 16$

$\mu_{\bar{x}} = \mu = 16$ ounces and $\sigma_{\bar{x}} = \sigma/\sqrt{n} = .18/\sqrt{16} = .045$ ounce

a. i. For $\bar{x} = 15.90$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.90 - 16)/.045 = -2.22$

For $\bar{x} = 15.95$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.95 - 16)/.045 = -1.11$

$P(15.90 < \bar{x} < 15.95) = P(-2.22 < z < -1.11) = P(z < -1.11) - P(z < -2.22) = .1335 - .0132$
$= .1203$

ii. For $\bar{x} = 15.95$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.95 - 16)/.045 = -1.11$

$P(\bar{x} < 15.95) = P(z < -1.11) = .1335$

iii. For $\bar{x} = 15.97$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.97 - 16)/.045 = -.67$

$P(\bar{x} > 15.97) = P(z > -.67) = 1 - P(z \le -.67) = 1 - .2514 = .7486$

b. $P(\bar{x}$ within .10 ounce of $\mu) = P(15.90 \le \bar{x} \le 16.10)$

For $\bar{x} = 15.90$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.90 - 16)/.045 = -2.22$

For $\bar{x} = 16.10$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (16.10 - 16)/.045 = 2.22$

$P(15.90 \le \bar{x} \le 16.10) = P(-2.22 \le z \le 2.22) = P(z \le 2.22) - P(z \le -2.22) = .9868 - .0132 = .9736$

c. $P(\bar{x}$ is less than μ by .135 ounce or more$) = P(\bar{x} < 15.865)$

For $\bar{x} = 15.865$: $z = (\bar{x} - \mu)/\sigma_{\bar{x}} = (15.865 - 16)/.045 = -3.00$

$P(\bar{x} < 15.865) = P(z < -3.00) = .0013$

18. $p = .048$, and $q = 1 - p = 1 - .048 = .952$

a. $n = 50$, $\mu_{\hat{p}} = p = .048$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.048)(.952)/50} = .0302$

$np = (50)(.048) = 2.4$ and $nq = (50)(.952) = 47.6$

Since $np < 5$, we can draw no conclusion about the shape of the sampling distribution of $\hat{p}$.

b. $n = 500$, $\mu_{\hat{p}} = p = .048$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.048)(.952)/500} = .0096$

$np = (500)(.048) = 24$ and $nq = (500)(.952) = 476$

Since $np > 5$ and $nq > 5$, the sampling distribution of $\hat{p}$ is approximately normal.

c. $n = 5000$, $\mu_{\hat{p}} = p = .048$, and $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.048)(.952)/5000} = .0030$

$np = (5000)(.048) = 240$ and $nq = (5000)(.952) = 4760$

Since $np > 5$ and $nq > 5$, the sampling distribution of $\hat{p}$ is approximately normal.

19. $p = .0352$, $q = 1 - p = 1 - .0352 = .9648$, and $n = 900$

a. $\sigma_{\hat{p}} = \sqrt{pq/n} = \sqrt{(.0352)(.9648)/900} = .00614283$

i. For $\hat{p} = .05$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.05 - .0352)/.00614283 = 2.41$

$P(\hat{p} > .05) = P(z > 2.41) = 1 - P(z \le 2.41) = 1 - .9920 = .0080$

ii. For $\hat{p} = .03$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.03 - .0352)/.00614283 = -.85$

For $\hat{p} = .0375$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.0375 - .0352)/.00614283 = .37$

$P(.03 < \hat{p} < .0375) = P(-.85 < z < .37) = P(z < .37) - P(z < -.85) = .6443 - .1977 = .4466$

iii. For $\hat{p} = .04$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.04 - .0352)/.00614283 = .78$

$P(\hat{p} < .04) = P(z < .78) = .7823$

iv. For $\hat{p} = .025$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.025 - .0352)/.00614283 = -1.66$

For $\hat{p} = .0325$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.0325 - .0352)/.00614283 = -.44$

$P(.025 < \hat{p} < .0325) = P(-1.66 < z < -.44) = P(z < -.44) - P(z < -1.66) = .3300 - .0485 = .2815$

b. $P(\hat{p}$ within .005 of $p) = P(.0302 \le \hat{p} \le .0402)$

For $\hat{p} = .0302$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.0302 - .0352)/.00614283 = -.81$

For $\hat{p} = .0402$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.0402 - .0352)/.00614283 = .81$

$P(.0302 \le \hat{p} \le .0402) = P(-.81 \le z \le .81) = P(z \le .81) - P(z \le -.81) = .7910 - .2090 = .5820$

c. $P(\hat{p}$ not within .008 of $p) = P(\hat{p} \le .0272) + P(\hat{p} \ge .0432)$

For $\hat{p} = .0272$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.0272 - .0352)/.00614283 = -1.30$

For $\hat{p} = .0432$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.0432 - .0352)/.00614283 = 1.30$

$P(\hat{p} \le .0272) + P(\hat{p} \ge .0432) = 1 - P(.0272 < \hat{p} < .0432) = 1 - P(-1.30 \le z \le 1.30)$

$= 1 - [P(z \le 1.30) - P(z \le -1.30)] = 1 - [.9032 - .0968] = 1 - .8064 = .1936$

d. $P(\hat{p}$ greater than p by .0095 or more$) = P(\hat{p} \geq .0447)$

For $\hat{p} = .0447$: $z = (\hat{p} - p)/\sigma_{\hat{p}} = (.0447 - .0352)/.00614283 = 1.55$

$P(\hat{p} \geq .0447) = P(z \geq 1.55) = 1 - P(z \leq 1.55) = 1 - .9394 = .0606$