Chapter Six

Sections 6.1 - 6.3

6.1 The probability distribution of a discrete random variable assigns probabilities to points while that of a continuous random variable assigns probabilities to intervals.

6.2 The probability that a continuous random variable x assumes a single value is always zero, that is, $P(x = a) = 0$.

6.3 Since $P(a) = 0$ and $P(b) = 0$ for a continuous random variable, $P(a \le x \le b) = P(a < x < b)$.

6.4 The following are the three main characteristics of a normal distribution.

1. The total area under a normal curve is 1.0.
2. A normal curve is symmetric about the mean. Consequently, 50% of the total area under a normal distribution curve lies on the left side of the mean, and 50% lies on the right side of the mean.
3. The tails of a normal distribution curve extend indefinitely in both directions without touching or crossing the horizontal axis. Although a normal curve never meets the horizontal axis, beyond the points represented by $\mu - 3\sigma$ to $\mu + 3\sigma$ it becomes so close to this axis that the area under the curve beyond these points in both directions is very close to zero.

These three characteristics are described with graphs on pages 257 and 258 of the text.

6.5 The **standard normal distribution** is a special case of the normal distribution. For the standard normal distribution, the value of the mean is equal to zero and the value of the standard deviation is 1. The units of the standard normal distribution curve are denoted by z and are called the z values or *z scores*. The *z values* on the right side of the mean (which is zero) are positive and those on the left side are negative. A specific value of z gives the distance between the mean and the point represented by z in terms of the standard deviation.

6.6 The parameters of the normal distribution are the mean μ and the standard deviation σ.

6.7 As its standard deviation decreases, the width of a normal distribution curve decreases and its height increases.

6.8 The width and height of a normal distribution do not change when its standard deviation remains the same but its mean increases.

6.9 For a standard normal distribution, the *z value* gives the distance between the mean and the point represented by z in terms of the standard deviation. The *z values* on the right side of the mean are positive and those on the left side are negative.

6.10 Area between $\mu - \sigma$ and $\mu + \sigma$ is the area between $z = -1$ and $z = 1$. Then

$P(-1 < z < 1) = P(z < 1) - P(z < -1) = .8413 - .1587 = .6826$

6.11 Area between $\mu - 1.5\sigma$ and $\mu + 1.5\sigma$ is the area between $z = -1.5$ and $z = 1.5$. Then,

$P(-1.5 < z < 1.5) = P(z < 1.5) - P(z < -1.5) = .9332 - .0668 = .8664$

6.12 Area within two standard deviations of the mean is:

$P(-2 < z < 2) = P(z < 2) - P(z < -2) = .9772 - .0228 = .9544$

6.13 Area within 2.5 standard deviations of the mean is:

$P(-2.5 < z < 2.5) = P(z < 2.5) - P(z < -2.5) = .9938 - .0062 = .9876$

6.14 Area within three standard deviations of the mean is:

$P(-3 < z < 3) = P(z < 3) - P(z < -3) = .9987 - .0013 = .9974$

6.15
a. $P(0 < z < 1.95) = P(z < 1.95) - P(z < 0) = .9744 - .5000 = .4744$
b. $P(-2.05 < z < 0) = P(z < 0) - P(z < -2.05) = .5000 - .0202 = .4798$
c. $P(1.15 < z < 2.37) = P(z < 2.37) - P(z < 1.15) = .9911 - .8749 = .1162$
d. $P(-2.88 \le z \le -1.53) = P(z \le -1.53) - P(z \le -2.88) = .0630 - .0020 = .0610$
e. $P(-1.67 \le z \le 2.24) = P(z \le 2.24) - P(z \le -1.67) = .9875 - .0475 = .9400$

6.16
a. $P(0 \le z \le 2.34) = P(z \le 2.34) - P(z \le 0) = .9904 - .5000 = .4904$
b. $P(-2.58 < z < 0) = P(z < 0) - P(z < -2.58) = .5000 - .0049 = .4951$
c. $P(.84 \le z \le 1.95) = P(z \le 1.95) - P(z \le .84) = .9744 - .7995 = .1749$
d. $P(-2.49 < z < -.57) = P(z < -.57) - P(z < -2.49) = .2843 - .0064 = .2779$
e. $P(-2.15 < z < 1.87) = P(z < 1.87) - P(z < -2.15) = .9693 - .0158 = .9535$

6.17
a. $P(z > 1.36) = 1 - P(z \le 1.36) = 1 - .9131 = .0869$
b. $P(z < -1.97) = .0244$
c. $P(z > -2.05) = 1 - P(z \le -2.05) = 1 - .0202 = .9798$
d. $P(z < 1.76) = .9608$

6.18
a. $P(z > 1.43) = 1 - P(z \le 1.43) = 1 - .9236 = .0764$
b. $P(z < -1.65) = .0495$

c. $P(z > -.65) = 1 - P(z \le -.65) = 1 - .2578 = .7422$

d. $P(z < .89) = .8133$

6.19 a. $P(0 < z < 4.28) = P(z < 4.28) - P(z < 0) = 1 - .5 = .5$ approximately

b. $P(0 \le z \le -3.75) = P(z \le 0) - P(z \le -3.75) = .5 - 0 = .5$ approximately

c. $P(z > 7.43) = 1 - P(z \le 7.43) = 1 - 1 = 0$ approximately

d. $P(z < -4.69) = 0$ approximately

6.20 a. $P(0 \le z \le 3.94) = P(z \le 3.94) - P(z \le 0) = 1 - .5 = .5$ approximately

b. $P(-5.16 < z < 0) = P(z < 0) - P(z < -5.16) = .5 - 0 = .5$ approximately

c. $P(z > 5.42) = 1 - P(z \le 5.42) = 1 - 1 = 0$ approximately

d. $P(z < -3.68) = 0$ approximately

6.21 a. $P(-1.83 \le z \le 2.57) = P(z \le 2.57) - P(z \le -1.83) = .9949 - .0336 = .9613$

b. $P(0 \le z \le 2.02) = P(z \le 2.02) - P(z \le 0) = .9783 - .5000 = .4783$

c. $P(-1.99 \le z \le 0) = P(z \le 0) - P(z \le -1.99) = .5000 - .0233 = .4767$

d. $P(z \ge 1.48) = 1 - P(z \le 1.48) = 1 - .9306 = .0694$

6.22 a. $P(-2.46 \le z \le 1.88) = P(z \le 1.88) - P(z \le -2.46) = .9699 - .0069 = .9630$

b. $P(0 \le z \le 1.96) = P(z \le 1.96) - P(z \le 0) = .9750 - .5000 = .4750$

c. $P(-2.58 \le z \le 0) = P(z \le 0) - P(z \le -2.58) = .5000 - .0049 = .4951$

d. $P(z \ge .73) = 1 - P(z \le .73) = 1 - .7673 = .2327$

6.23 a. $P(z < -2.34) = .0096$

b. $P(.67 \le z \le 2.59) = P(z \le 2.59) - P(z \le .67) = .9952 - .7486 = .2466$

c. $P(-2.07 \le z \le -.93) = P(z \le -.93) - P(z \le -2.07) = .1762 - .0192 = .1570$

d. $P(z < 1.78) = .9625$

6.24 a. $P(z < -1.31) = .0951$

b. $P(1.23 \le z \le 2.89) = P(z \le 2.89) - P(z \le 1.23) = .9981 - .8907 = .1074$

c. $P(-2.24 \le z \le -1.19) = P(z \le -1.19) - P(z \le -2.24) = .1170 - .0125 = .1045$

d. $P(z < 2.02) = .9783$

6.25 a. $P(z > -.98) = 1 - P(z \le -.98) = 1 - .1635 = .8365$

b. $P(-2.47 \le z \le 1.29) = P(z \le 1.29) - P(z \le -2.47) = .9015 - .0068 = .8947$

c. $P(0 \le z \le 4.25) = P(z \le 4.25) - P(z \le 0) = 1 - .5 = .5$ approximately

d. $P(-5.36 \le z \le 0) = P(z \le 0) - P(z \le -5.36) = .5 - 0 = .5$ approximately

e. $P(z > 6.07) = 1 - P(z \le 6.07) = 1 - 1 = 0$ approximately

f. $P(z < -5.27) = 0$ approximately

6.26
a. $P(z > -1.86) = 1 - P(z \le -1.86) = 1 - .0314 = .9686$
b. $P(-.68 \le z \le 1.94) = P(z \le 1.94) - P(z \le -.68) = .9738 - .2483 = .7255$
c. $P(0 \le z \le 3.85) = P(z \le 3.85) - P(z \le 0) = 1 - .5 = .5$ approximately
d. $P(-4.34 \le z \le 0) = P(z \le 0) - P(z \le -4.34) = .5 - 0 = .5$ approximately
e. $P(z > 4.82) = 1 - P(z \le 4.82) = 1 - 1 = 0$ approximately
f. $P(z < -6.12) = 0$ approximately

Section 6.4

6.27 $\mu = 30$ and $\sigma = 5$
a. $z = (x - \mu)/\sigma = (39 - 30)/5 = 1.80$
b. $z = (x - \mu)/\sigma = (19 - 30)/5 = -2.20$
c. $z = (x - \mu)/\sigma = (24 - 30)/5 = -1.20$
d. $z = (x - \mu)/\sigma = (44 - 30)/5 = 2.80$

6.28 $\mu = 16$ and $\sigma = 3$
a. $z = (x - \mu)/\sigma = (12 - 16)/3 = -1.33$
b. $z = (x - \mu)/\sigma = (22 - 16)/3 = 2.00$
c. $z = (x - \mu)/\sigma = (19 - 16)/3 = 1.00$
d. $z = (x - \mu)/\sigma = (13 - 16)/3 = -1.00$

6.29 $\mu = 20$ and $\sigma = 4$
a. For $x = 20$: $z = (x - \mu)/\sigma = (20 - 20)/4 = 0$
For $x = 27$: $z = (x - \mu)/\sigma = (27 - 20)/4 = 1.75$
$P(20 < x < 27) = P(0 < z < 1.75) = P(z < 1.75) - P(z < 0) = .9599 - .5000 = .4599$
b. For $x = 23$: $z = (x - \mu)/\sigma = (23 - 20)/4 = .75$
For $x = 26$: $z = (x - \mu)/\sigma = (26 - 20)/4 = 1.50$
$P(23 \le x \le 26) = P(.75 \le z \le 1.50) = P(z \le 1.50) - P(z \le .75) = .9332 - .7734 = .1598$
c. For $x = 9.5$: $z = (x - \mu)/\sigma = (9.5 - 20)/4 = -2.63$
For $x = 17$: $z = (x - \mu)/\sigma = (17 - 20)/4 = -.75$
$P(9.5 < x < 17) = P(-2.63 < z < -.75) = P(z < -.75) - P(z < -2.63) = .2266 - .0043 = .2223$

6.30 $\mu = 12$ and $\sigma = 2$
a. For $x = 7.76$: $z = (x - \mu)/\sigma = (7.76 - 12)/2 = -2.12$
For $x = 12$: $z = (x - \mu)/\sigma = (12 - 12)/2 = 0$
$P(7.76 < x < 12) = P(-2.12 < z < 0) = P(z < 0) - P(z < -2.12) = .5000 - .0170 = .4830$
b. For $x = 14.48$: $z = (x - \mu)/\sigma = (14.48 - 12)/2 = 1.24$
For $x = 16.54$: $z = (x - \mu)/\sigma = (16.54 - 12)/2 = 2.27$
$P(14.48 < x < 16.45) = P(1.24 < z < 2.27) = P(z < 2.27) - P(z < 1.24) = .9884 - .8925 = .0959$

c. For $x = 8.22$: $z = (x - \mu)/\sigma = (8.22 - 12)/2 = -1.89$

For $x = 10.06$: $z = (x - \mu)/\sigma = (10.06 - 12)/2 = -.97$

$P(8.22 \le x \le 10.06) = P(-1.89 \le z \le -.97) = P(z \le -.97) - P(z \le -1.89) = .1660 - .0294 = .1366$

6.31 $\mu = 55$ and $\sigma = 7$

a. For $x = 58$: $z = (x - \mu)/\sigma = (58 - 55)/7 = .43$

$P(x > 58) = P(z > .43) = 1 - P(z \le .43) = 1 - .6664 = .3336$

b. For $x = 43$: $z = (x - \mu)/\sigma = (43 - 55)/7 = -1.71$

$P(x > 43) = P(z > -1.71) = 1 - P(z \le -1.71) = 1 - .0436 = .9564$

c. For $x = 68$: $z = (x - \mu)/\sigma = (68 - 55)/7 = 1.86$

$P(x < 68) = P(z < 1.86) = .9686$

d. For $x = 22$: $z = (x - \mu)/\sigma = (22 - 55)/7 = -4.71$

$P(x < 22) = P(z < -4.71) = 0$ approximately

6.32 $\mu = 37$ and $\sigma = 3$

a. For $x = 30$: $z = (x - \mu)/\sigma = (30 - 37)/3 = -2.33$

$P(x < 30) = P(z < -2.33) = .0099$

b. For $x = 52$: $z = (x - \mu)/\sigma = (52 - 37)/3 = 5.00$

$P(x > 52) = P(z > 5.00) = 1 - P(z \le 5.00) = 1 - 1 = 0$ approximately

c. For $x = 44$: $z = (x - \mu)/\sigma = (44 - 37)/3 = 2.33$

$P(x < 44) = P(z < 2.33) = .9901$

d. For $x = 32$: $z = (x - \mu)/\sigma = (32 - 37)/3 = -1.67$

$P(x > 32) = P(z > -1.67) = 1 - P(z \le -1.67) = 1 - .0475 = .9525$

6.33 $\mu = 25$ and $\sigma = 6$

a. For $x = 29$: $z = (x - \mu)/\sigma = (29 - 25)/6 = .67$

For $x = 36$: $z = (x - \mu)/\sigma = (36 - 25)/6 = 1.83$

$P(29 < x < 36) = P(.67 < z < 1.83) = P(z < 1.83) - P(z < .67) = .9664 - .7486 = .2178$

b. For $x = 22$: $z = (x - \mu)/\sigma = (22 - 25)/6 = -.50$

For $x = 35$: $z = (x - \mu)/\sigma = (35 - 25)/6 = 1.67$

$P(22 < x < 35) = P(-.50 < z < 1.67) = P(z < 1.67) - P(z < -.50) = .9525 - .3085 = .6440$

6.34 $\mu = 40$ and $\sigma = 4$

a. For $x = 29$: $z = (x - \mu)/\sigma = (29 - 40)/4 = -2.75$

For $x = 35$: $z = (x - \mu)/\sigma = (35 - 40)/4 = -1.25$

$P(29 < x < 35) = P(-2.75 < z < -1.25) = P(z < -1.25) - P(z < -2.75) = .1056 - .0030 = .1026$

b. For $x = 34$: $z = (x - \mu)/\sigma = (34 - 40)/4 = -1.50$

For $x = 50$: $z = (x - \mu)/\sigma = (50 - 40)/4 = 2.50$

$P(34 \le x \le 50) = P(-1.50 \le z \le 2.50) = P(z \le 2.50) - P(z \le -1.50) = .9938 - .0668 = .9270$

6.35 $\mu = 80$ and $\sigma = 12$

a. For $x = 69$: $z = (x - \mu)/\sigma = (69 - 80)/12 = -.92$

$P(x > 69) = P(z > -.92) = 1 - P(z \le -.92) = 1 - .1788 = .8212$

b. For $x = 73$: $z = (x - \mu)/\sigma = (73 - 80)/12 = -.58$

$P(x < 73) = P(z < -.58) = .2810$

c. For $x = 101$: $z = (x - \mu)/\sigma = (101 - 80)/12 = 1.75$

$P(x > 101) = P(z > 1.75) = 1 - P(z \le 1.75) = 1 - .9599 = .0401$

d. For $x = 87$: $z = (x - \mu)/\sigma = (87 - 80)/12 = .58$

$P(x < 87) = P(z < .58) = .7190$

6.36 $\mu = 65$ and $\sigma = 15$

a. For $x = 45$: $z = (x - \mu)/\sigma = (45 - 65)/15 = -1.33$

$P(x < 45) = P(z < -1.33) = .0918$

b. For $x = 79$: $z = (x - \mu)/\sigma = (79 - 65)/15 = .93$

$P(x > 79) = P(z > .93) = 1 - P(z \le .93) = 1 - .8238 = .1762$

c. For $x = 54$: $z = (x - \mu)/\sigma = (54 - 65)/15 = -.73$

$P(x > 54) = P(z > -.73) = 1 - P(z \le -.73) = 1 - .2327 = .7673$

d. For $x = 70$: $z = (x - \mu)/\sigma = (70 - 65)/15 = .33$

$P(x < 70) = P(z < .33) = .6293$

Section 6.5

6.37 $\mu = 190$ minutes and $\sigma = 21$ minutes

a. For $x = 160$: $z = (x - \mu)/\sigma = (160 - 190)/21 = -1.43$

$P(x < 160) = P(z < -1.43) = .0764$

b. For $x = 215$: $z = (x - \mu)/\sigma = (215 - 190)/21 = 1.19$

For $x = 245$: $z = (x - \mu)/\sigma = (245 - 190)/21 = 2.62$

$P(215 \le x \le 245) = P(1.19 \le z \le 2.62) = P(z \le 2.62) - P(z \le 1.19) = .9956 - .8830 = .1126$

6.38 $\mu = 4.4$ hours and $\sigma = 1.08$ hour

a. For $x = 3.0$: $z = (x - \mu)/\sigma = (3.0 - 4.4)/1.08 = -1.30$

For $x = 5.0$: $z = (x - \mu)/\sigma = (5.0 - 4.4)/1.08 = .56$

$P(3.0 < x < 5.0) = P(-1.30 < z < .56) = P(z < .56) - P(z < -1.30) = .7123 - .0968 = .6155$

b. For $x = 2.0$: $z = (x - \mu)/\sigma = (2.0 - 4.4)/1.08 = -2.22$

$P(x < 2.0) = P(z < -2.22) = .0132$

6.39 $\mu = 9.32$ and $\sigma = 1.38$

a. For $x = 11.1$: $z = (x - \mu)/\sigma = (11.1 - 9.32)/1.38 = 1.29$

$P(x > 11.1) = P(z > 1.29) = 1 - P(z \le 1.29) = 1 - .9015 = .0985$

b. For $x = 6.0$: $z = (x - \mu)/\sigma = (6.0 - 9.32)/1.38 = -2.41$

For $x = 7.2$: $z = (x - \mu)/\sigma = (7.2 - 9.32)/1.38 = -1.54$

$P(6.0 < x < 7.2) = P(-2.41 < z < -1.54) = P(z < -1.54) - P(z < -2.41) = .0618 - .0080 = .0538$

6.40 $\mu = 36$ seconds and $\sigma = 2.5$ seconds

a. For $x = 39$: $z = (x - \mu)/\sigma = (39 - 36)/2.5 = 1.20$

$P(x > 39) = P(z > 1.20) = 1 - P(z \le 1.20) = 1 - .8849 = .1151$ or 11.51%

b. For $x = 29$: $z = (x - \mu)/\sigma = (29 - 36)/2.5 = -2.80$

For $x = 34$: $z = (x - \mu)/\sigma = (34 - 36)/2.5 = -.80$

$P(29 < x < 34) = P(-2.80 < z < -.80) = P(z < -.80) - P(z < -2.80) = .2119 - .0026 = .2093$ or 20.93%

6.41 $\mu = 46$ miles per hour and $\sigma = 4$ miles per hour

a. For $x = 40$: $z = (x - \mu)/\sigma = (40 - 46)/4 = -1.50$

$P(x > 40) = P(z > -1.50) = 1 - P(z \le -1.50) = 1 - .0668 = .9332$ or 93.32%

b. For $x = 50$: $z = (x - \mu)/\sigma = (50 - 46)/4 = 1.00$

For $x = 57$: $z = (x - \mu)/\sigma = (57 - 46)/4 = 2.75$

$P(50 < x < 57) = P(1.00 < z < 2.75) = P(z < 2.75) - P(z < 1.00) = .9970 - .8413 = .1557$ or 15.57%

6.42 $\mu = \$845$ and $\sigma = \$270$

a. For $x = 1000$: $z = (x - \mu)/\sigma = (1000 - 845)/270 = .57$

For $x = 1440$: $z = (x - \mu)/\sigma = (1440 - 845)/270 = 2.20$

$P(1000 < x < 1440) = P(.57 < z < 2.20) = P(z < 2.20) - P(z < .57) = .9861 - .7157 = .2704$

b. For $x = 730$: $z = (x - \mu)/\sigma = (730 - 845)/270 = -.43$

$P(x \ge 730) = P(z \ge -.43) = 1 - P(z \le -.43) = 1 - .3336 = .6664$ or 66.64%

6.43 $\mu = 190$ minutes and $\sigma = 53.4$ minutes

a. For $x = 300$: $z = (x - \mu)/\sigma = (300 - 190)/53.4 = 2.06$

$P(x > 300) = P(z > 2.06) = 1 - P(z \le 2.06) = 1 - .9803 = .0197$

b. For $x = 120$: $z = (x - \mu)/\sigma = (120 - 190)/53.4 = -1.31$

For $x = 180$: $z = (x - \mu)/\sigma = (180 - 190)/53.4 = -.19$

$P(120 < x < 180) = P(-1.31 < z < -.19) = P(z < -.19) - P(z < -1.31) = .4247 - .0951 = .3296$

6.44 $\mu = 72{,}000$ miles and $\sigma = 13{,}000$ miles

a. For $x = 40{,}000$: $z = (x - \mu)/\sigma = (40{,}000 - 72{,}000)/13{,}000 = -2.46$

$P(x \le 40{,}000) = P(z \le -2.46) = .0069$ or .69%

b. For $x = 100{,}000$: $z = (x - \mu)/\sigma = (100{,}000 - 72{,}000)/13{,}000 = 2.15$

$P(x > 100{,}000) = P(z > 2.15) = 1 - P(z \le 2.15) = 1 - .9842 = .0158$ or 1.58%

6.45 $\mu = 1650$ kwh and $\sigma = 320$ kwh

a. For $x = 1950$: $z = (x - \mu)/\sigma = (1950 - 1650)/320 = .94$

$P(x < 1950) = P(z < .94) = .8264$

b. For $x = 900$: $z = (x - \mu)/\sigma = (900 - 1650)/320 = -2.34$

For $x = 1300$: $z = (x - \mu)/\sigma = (1300 - 1650)/320 = -1.09$

$P(900 \le z \le 1300) = P(-2.34 \le z \le -1.09) = P(z \le -1.09) - P(z \le -2.34) = .1379 - .0096 = .1283$ or 12.83%

6.46 $\mu = \$95$ and $\sigma = \$20$

For $x = 130$: $z = (x - \mu)/\sigma = (130 - 95)/20 = 1.75$

$P(x > 130) = P(z > 1.75) = 1 - P(z \le 1.75) = 1 - .9599 = .0401$ or 4.01%

6.47 $\mu = \$19{,}800$ and $\sigma = \$350$

a. For $x = 19{,}445$: $z = (x - \mu)/\sigma = (19{,}445 - 19{,}800)/350 = -1.01$

$P(x < 19{,}445) = P(z < -1.01) = .1562$ or 15.62%

b. For $x = 20{,}300$: $z = (x - \mu)/\sigma = (20{,}300 - 19{,}800)/350 = 1.43$

$P(x > 20{,}300) = P(z > 1.43) = 1 - P(z \le 1.43) = 1 - .9236 = .0764$ or 7.64%

6.48 $\mu = 7.59$ and $\sigma = .73$

a. For $x = 6$: $z = (x - \mu)/\sigma = (6 - 7.59)/.73 = -2.18$

$P(x < 6) = .P(z < -2.18) = .0146$ or 1.46%

b. For $x = 7$: $z = (x - \mu)/\sigma = (7 - 7.59)/.73 = -.81$

For $x = 8$: $z = (x - \mu)/\sigma = (8 - 7.59)/.73 = .56$

$P(7 < x < 8) = P(-.81 < z < .56) = P(z < .56) - P(z < -.81) = .7123 - .2090 = .5033$

c. For $x = 9$: $z = (x - \mu)/\sigma = (9 - 7.59)/.73 = 1.93$

$P(x \ge 9) = P(z \ge 1.93) = 1 - P(z \le 1.93) = 1 - .9732 = .0268$ or 2.68%

6.49 $\mu = 225$ minutes and $\sigma = 62$ minutes

a. For $x = 60$: $z = (x - \mu)/\sigma = (60 - 225)/62 = -2.66$

$P(x < 60) = P(z < -2.66) = .0039$ or .39%

b. For $x = 360$: $z = (x - \mu)/\sigma = (360 - 225)/62 = 2.18$

$P(x > 360) = P(z > 2.18) = 1 - P(z \le 2.18) = 1 - .9854 = .0146$ or 1.46%

c. For $x = 120$: $z = (x - \mu)/\sigma = (120 - 225)/62 = -1.69$

For $x = 180$: $z = (x - \mu)/\sigma = (180 - 225)/62 = -.73$

$P(120 \le x \le 180) = P(-1.69 \le z \le -.73) = P(z \le -.73) - P(z \le -1.69) = .2327 - .0455 = .1872$ or 18.72%

d. For $x = 240$: $z = (x - \mu)/\sigma = (240 - 225)/62 = .24$

For $x = 300$: $z = (x - \mu)/\sigma = (300 - 225)/62 = 1.21$

$P(240 \le x \le 300) = P(.24 \le z \le 1.21) = P(z \le 1.21) - P(z \le .24) = .8869 - .5948 = .2921$ or 29.21%

6.50 $\mu = 15$ minutes and $\sigma = 2.4$ minutes

a. For $x = 20$: $z = (x - \mu)/\sigma = (20 - 15)/2.4 = 2.08$

$P(x > 20) = P(z > 2.08) = 1 - P(z \le 2.08) = 1 - .9812 = .0188$ or 1.88%

b. For $x = 25$: $z = (x - \mu)/\sigma = (25 - 15)/2.4 = 4.17$

$P(x > 25) = P(z > 4.17) = 1 - P(z \le 4.17) = 1 - 1 = 0$ approximately

Although it is possible that a given car may take more than 25 minutes for oil and lube service, the probability is almost zero.

6.51 $\mu = 3.0$ inches and $\sigma = .009$ inch

For $x = 2.98$: $z = (x - \mu)/\sigma = (2.98 - 3.0)/.009 = -2.22$

For $x = 3.02$: $z = (x - \mu)/\sigma = (3.02 - 3.0)/.009 = 2.22$

$P(x < 2.98) + P(x > 3.02) = 1 - [P(2.98 \le x \le 3.02)] = 1 - [P(-2.22 \le z \le 2.22)]$

$= 1 - [P(z \le 2.22) - P(z \le -2.22)] = 1 - [.9868 - .0132] = 1 - .9736 = .0264$ or 2.64%

6.52 $\mu = 5.75$ ounces and $\sigma = .11$ ounce

For $x = 5.5$: $z = (x - \mu)/\sigma = (5.5 - 5.75)/.11 = -2.27$

For $x = 6.0$: $z = (x - \mu)/\sigma = (6.0 - 5.75)/.11 = 2.27$

$P(x < 5.5) + P(x > 6.0) = 1 - [P(5.5 \le x \le 6.0)] = 1 - [P(-2.27 \le z \le 2.27)]$

$= 1 - [P(z \le 2.27) - P(z \le -2.27)] = 1 - [.9884 - .0116] = 1 - .9768 = .0232$ or 2.32%

Section 6.6

6.53 a. $z = 2.00$ b. $z = -2.02$ approximately

c. $z = -.37$ approximately d. $z = 1.02$ approximately

6.54 a. $z = .51$ approximately b. $z = -.75$ approximately

c. $z = -.82$ approximately d. $z = 1.25$ approximately

6.55 a. $z = 1.65$ approximately b. $z = -1.96$

c. $z = -2.33$ approximately d. $z = 2.58$ approximately

6.56 a. $z = 1.96$ b. $z = -1.65$ approximately

c. $z = -3.08$ approximately d. $z = 2.33$ approximately

6.57 $\mu = 200$ and $\sigma = 25$

a. $z = .34$, $x = \mu + z\sigma = 200 + (.34)(25) = 208.50$

b. $z = 1.65$, $x = \mu + z\sigma = 200 + (1.65)(25) = 241.25$

c. $z = -.86$, $x = \mu + z\sigma = 200 + (-.86)(25) = 178.50$

d. $z = -2.17$, $x = \mu + z\sigma = 200 + (-2.17)(25) = 145.75$

e. $z = -1.67$, $x = \mu + z\sigma = 200 + (-1.67)(25) = 158.25$

f. $z = 2.05$, $x = \mu + z\sigma = 200 + (2.05)(25) = 251.25$

6.58 $\mu = 550$ and $\sigma = 75$

a. $z = -1.96$, $x = \mu + z\sigma = 550 + (-1.96)(75) = 403$

b. $z = -1.51$, $x = \mu + z\sigma = 550 + (-1.51)(75) = 436.75$

c. $z = 1.92$, $x = \mu + z\sigma = 550 + (1.92)(75) = 694$

d. $z = 1.75$, $x = \mu + z\sigma = 550 + (1.75)(75) = 681.25$

e. $z = -1.88$, $x = \mu + z\sigma = 550 + (-1.88)(75) = 409$

f. $z = 1.34$, $x = \mu + z\sigma = 550 + (1.34)(75) = 650.5$

6.59 $\mu = 15$ minutes and $\sigma = 2.4$ minutes

Let x denote the time to service a randomly chosen car. We are to find x so that the area in the right tail of the normal distribution curve is .05. Thus, $z = 1.65$ and $x = \mu + z\sigma = 15 + (1.65)(2.4)$ $= 18.96$ minutes. The maximum guaranteed waiting time should be approximately 19 minutes.

6.60 $\mu = \$95$ and $\sigma = \$20$

Let x denote the amount spent by a randomly chosen customer on a visit to this store. We are to find x such that the area in the right tail of the normal distribution curve is .10. Thus, $z = 1.28$ and $x = \mu + z\sigma = 95 + (1.28)(20) = \$120.60$. So, a required minimum purchase of \$121 would meet the condition.

6.61 $\mu = 1650$ kwh and $\sigma = 320$ kwh

Let x denote the amount of electric consumption during the winter by a randomly selected customer. We are to find x such that the area to the left of x in the normal distribution curve is .90. Thus, $z = 1.28$ and $x = \mu + z\sigma = 1650 + (1.28)(320) = 2059.6$ kwh. Bill Johnson's monthly electric consumption is approximately 2060 kwh.

6.62 $\mu = 70$ months and $\sigma = 8$ months

Let x denote the warranty period.

a. We are to find x such that the area in the left tail of the normal distribution curve is .01. Thus, $z = -2.33$ and $x = \mu + z\sigma = 70 + (-2.33)(8) = 51.36$ months. The warranty period should be approximately 51 months.

b. We are to find x such that the area in the left tail of the normal distribution curve is .05. Thus, $z = -1.65$ and $x = \mu + z\sigma = 70 + (-1.65)(8) = 56.8$ months. The warranty period should be approximately 56 months.

6.63 σ = \$9.50 and $P(x \geq 90) = .20$

The area to the left of $x = 90$ is $1 - .20 = .80$ and $z = .84$ approximately. Then, from $x = \mu + z\sigma$ obtain $\mu = x - z\sigma = 90 - (.84)(9.50) = \$82.02$. The mean price of all college textbooks is approximately \$82.

6.64 $\sigma = .35$ and $P(x \geq 64) = .95$

The area to the left of $x = 64$ is $1 - .95 = .05$ and $z = -1.65$ approximately. Then, $\mu = x - z\sigma$ $= 64 - (-1.65)(.35) = 64.58$ ounces. The mean amount of detergent poured by this machine into jugs should be approximately 65 ounces.

Section 6.7

6.65 The normal distribution may be used as an approximation to a binomial distribution when both $np > 5$ and $nq > 5$.

6.66 a. From Table I of Appendix C, for $n = 20$ and $p = .60$, $P(x = 14) = .1244$

b. $\mu = np = 20(.60) = 12$ and $\sigma = \sqrt{npq} = \sqrt{20(.60)(.40)} = 2.19089023$

For $x = 13.5$: $z = (13.5 - 12)/2.19089023 = .68$

For $x = 14.5$: $z = (14.5 - 12)/2.19089023 = 1.14$

$P(13.5 \leq x \leq 14.5) = P(.68 \leq z \leq 1.14) = P(z \leq 1.14) - P(z \leq .68) = .8729 - .7517 = .1212$

The difference between this approximation and the exact probability is $.1244 - .1212 = .0032$

6.67 a. From Table I of Appendix C, for $n = 25$ and $p = .40$,

$P(8 \leq x \leq 13) = P(x = 8) + P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12) + P(x = 13)$

$= .1200 + .1511 + .1612 + .1465 + .1140 + .0760 = .7688$

b. $\mu = np = 25(.40) = 10$ and $\sigma = \sqrt{npq} = \sqrt{25(.40)(.60)} = 2.44948974$

For $x = 7.5$: $z = (7.5 - 10)/2.44948974 = -1.02$

For $x = 13.5$: $z = (13.5 - 10)/2.44948974 = 1.43$

$P(7.5 \leq x \leq 13.5) = P(-1.02 \leq z \leq 1.43) = P(z \leq 1.43) - P(z \leq -1.02) = .9236 - .1539 = .7697$

The difference between this approximation and the exact probability is $.7697 - .7688 = .0009$

6.68 a. $\mu = np = 80(.50) = 40$ and $\sigma = \sqrt{npq} = \sqrt{80(.50)(.50)} = 4.47213596$

b. For $x = 41.5$: $z = (41.5 - 40)/4.47213596 = .34$

$P(x \geq 41.5) = P(z \geq .34) = 1 - P(z \leq .34) = 1 - .6331 = .3669$

c. For $x = 40.5$: $z = (40.5 - 40)/4.47213596 = .11$

For $x = 48.5$: $z = (48.5 - 40)/4.47213596 = 1.90$

$P(40.5 \leq x \leq 48.5) = P(.11 \leq z \leq 1.90) = P(z \leq 1.90) - P(z \leq .11) = .9713 - .5438 = .4275$

6.69 a. $\mu = np = 120(.60) = 72$ and $\sigma = \sqrt{npq} = \sqrt{120(.60)(.40)} = 5.36656315$

b. For $x = 69.5$: $z = (69.5 - 72)/5.36656315 = -.47$

$P(x \le 69.5) = P(z \le -.47) = .3192$

c. For $x = 66.5$: $z = (66.5 - 72)/5.36656315 = -1.02$

For $x = 73.5$: $z = (73.5 - 72)/5.36656315 = .28$

$P(66.5 \le x \le 73.5) = P(-1.02 \le z \le .28) = P(z \le .28) - P(z \le -1.02) = .6103 - .1539 = .4564$

6.70 a. $\mu = np = 140(.45) = 63$ and $\sigma = \sqrt{npq} = \sqrt{140(.45)(.55)} = 5.88642506$

For $x = 66.5$: $z = (66.5 - 63)/5.88642506 = .59$

For $x = 67.5$: $z = (67.5 - 63)/5.88642506 = .76$

$P(66.5 \le x \le 67.5) = P(.59 \le z \le .76) = P(z \le .76) - P(z \le .59) = .7764 - .7224 = .0540$

b. $\mu = np = 100(.55) = 55$ and $\sigma = \sqrt{npq} = \sqrt{100(.55)(.45)} = 4.97493719$

For $x = 51.5$: $z = (51.5 - 55)/4.97493719 = -.70$

For $x = 60.5$: $z = (60.5 - 55)/4.97493719 = 1.11$

$P(51.5 \le x \le 60.5) = P(-.70 \le z \le 1.11) = P(z \le 1.11) - P(z \le -.70) = .8665 - .2420 = .6245$

c. $\mu = np = 90(.42) = 37.8$ and $\sigma = \sqrt{npq} = \sqrt{90(.42)(.58)} = 4.68230712$

For $x = 39.5$: $z = (39.5 - 37.80)/4.68230712 = .36$

$P(x \ge 39.5) = P(z \ge .36) = 1 - P(z \le .36) = 1 - .6406 = .3594$

d. $\mu = np = 104(.75) = 78$ and $\sigma = \sqrt{npq} = \sqrt{104(.75)(.25)} = 4.41588043$

For $x = 72.5$: $z = (72.5 - 78)/4.41588043 = -1.25$

$P(x \le 72.5) = P(z \le -1.25) = .1056$

6.71 a. $\mu = np = 70(.30) = 21$ and $\sigma = \sqrt{npq} = \sqrt{70(.30)(.70)} = 3.83405790$

For $x = 17.5$: $z = (17.5 - 21)/3.83405790 = -.91$

For $x = 18.5$: $z = (18.5 - 21)/3.83405790 = -.65$

$P(17.5 \le x \le 18.5) = P(-.91 \le z \le -.65) = P(z \le -.65) - P(z \le -.91) = .2578 - .1814 = .0764$

b. $\mu = np = 200(.70) = 140$ and $\sigma = \sqrt{npq} = \sqrt{200(.70)(.30)} = 6.48074070$

For $x = 132.5$: $z = (132.5 - 140)/6.48074070 = -1.16$

For $x = 145.5$: $z = (145.5 - 140)/6.48074070 = .85$

$P(132.5 \le x \le 145.5) = P(-1.16 \le z \le .85) = P(z \le .85) - P(z \le -1.16) = .8023 - .1230 = .6793$

c. $\mu = np = 85(.40) = 34$ and $\sigma = \sqrt{npq} = \sqrt{85(.40)(.60)} = 4.51663592$

For $x = 29.5$: $z = (29.5 - 34)/4.51663592 = -1.00$

$P(x \ge 29.5) = P(z \ge -1.00) = 1 - P(z \le -1.00) = 1 - .1587 = .8413$

d. $\mu = np = 150(.38) = 57$ and $\sigma = \sqrt{npq} = \sqrt{150(.38)(.62)} = 5.94474558$

For $x = 62.5$: $z = (62.5 - 57)/5.94474558 = .93$

$P(x \le 62.5) = P(z \le .93) = .8238$

6.72 $\mu = np = 500(.78) = 390$ and $\sigma = \sqrt{npq} = \sqrt{500(.78)(.22)} = 9.26282894$

For $x = 374.5$: $z = (374.5 - 390)/9.26282894 = -1.67$

For $x = 385.5$: $z = (385.5 - 390)/9.26282894 = -.49$

$P(374.5 \le x \le 385.5) = P(-1.67 \le z \le -.49) = P(z \le -.49) - P(z \le -1.67) = .3121 - .0475 = .2646$

6.73 $\mu = np = 750(.048) = 36$ and $\sigma = \sqrt{npq} = \sqrt{750(.048)(.952)} = 5.85422924$

For $x = 44.5$: $z = (44.5 - 36)/5.85422924 = 1.45$

$P(x \ge 44.5) = P(z \ge 1.45) = 1 - P(z \le 1.45) = 1 - .9265 = .0735$

6.74 $\mu = np = 600(.0352) = 21.12$ and $\sigma = \sqrt{npq} = \sqrt{600(.0352)(.9648)} = 4.51404209$

a. For $x = 24.5$: $z = (24.5 - 21.12)/4.51404209 = .75$

For $x = 25.5$: $z = (25.5 - 21.12)/4.51404209 = .97$

$P(24.5 \le x \le 25.5) = P(.75 \le z \le .97) = P(z \le .97) - P(z \le .75) = .8340 - .7734 = .0606$

b. For $x = 12.5$: $z = (12.5 - 21.12)/4.51404209 = -1.91$

For $x = 19.5$: $z = (19.5 - 21.12)/4.51404209 = -.36$

$P(12.5 \le x \le 19.5) = P(-1.91 \le z \le -.36) = P(z \le -.36) - P(z \le -1.91) = .3594 - .0281 = .3313$

c. For $x = 26.5$: $z = (26.5 - 21.12)/4.51404209 = 1.19$

$P(x \ge 26.5) = P(z \ge 1.19) = 1 - P(z \le 1.19) = 1 - .8830 = .1170$

6.75 $\mu = np = 400(.29) = 116$ and $\sigma = \sqrt{npq} = \sqrt{400(.29)(.71)} = 9.07524104$

a. For $x = 109.5$: $z = (109.5 - 116)/9.07524104 = -.72$

For $x = 110.5$: $z = (110.5 - 116)\ 9.07524104 = -.61$

$P(109.5 \le x \le 110.5) = P(-.72 \le z \le -.61) = P(z \le -.61) - P(z \le -.72) = .2709 - .2358 = .0351$

b. For $x = 123.5$: $z = (123.5 - 116)/9.07524104 = .83$

For $x = 135.5$: $z = (135.5 - 116)/9.07524104 = 2.15$

$P(123.5 \le x \le 135.5) = P(.83 \le z \le 2.15) = P(z \le 2.15) - P(z \le .83) = .9842 - .7967 = .1875$

c. For $x = 105.5$: $z = (105.5 - 116)/9.07524104 = -1.16$

$P(x \le 105.5) = P(z \le -1.16) = .1230$

6.76 $\mu = np = 250(.202) = 50.5$ and $\sigma = \sqrt{npq} = \sqrt{250(.202)(.798)} = 6.34814934$

a. For $x = 56.5$: $z = (56.5 - 50.5)/6.34814934 = .95$

For $x = 57.5$: $z = (57.5 - 50.5)/6.34814934 = 1.10$

$P(56.5 \le x \le 57.5) = P(.95 \le z \le 1.10) = P(z \le 1.10) - P(z \le .95) = .8643 - .8289 = .0354$

b. For $x = 34.5$: $z = (34.5 - 50.5)/6.34814934 = -2.52$

For $x = 41.5$: $z = (41.5 - 50.5)/6.34814934 = -1.42$

$P(34.5 \le x \le 41.5) = P(-2.52 \le z \le -1.42) = P(z \le -1.42) - P(z \le -2.52) = .0778 - .0059 = .0719$

c. For $x = 59.5$: $z = (59.5 - 50.5)/6.34814934 = 1.42$

$P(x \ge 59.5) = P(z \ge 1.42) = 1 - P(z \le 1.42) = 1 - .9222 = .0778$

6.77 $\mu = np = 100(.80) = 80$ and $\sigma = \sqrt{npq} = \sqrt{100(.80)(.20)} = 4$

a. For $x = 74.5$: $z = (74.5 - 80)/4 = -1.38$

For $x = 75.5$: $z = (75.5 - 80)/4 = -1.13$

$P(74.5 \le x \le 75.5) = P(-1.38 \le z \le -1.13) = P(z \le -1.13) - P(z \le -1.38) = .1292 - .0838 = .0454$

b. For $x = 73.5$: $z = (73.5 - 80)/4 = -1.63$

$P(x \le 73.5) = P(z \le -1.63) = .0516$

c. For $x = 73.5$: $z = (73.5 - 80)/4 = -1.63$

For $x = 85.5$: $z = (85.5 - 80)/4 = 1.38$

$P(73.5 \le x \le 85.5) = P(-1.63 \le z \le 1.38) = P(z \le 1.38) - P(z \le -1.63) = .9162 - .0516 = .8646$

6.78 $\mu = np = 500(.05) = 25$ and $\sigma = \sqrt{npq} = \sqrt{500(.05)(.95)} = 4.87339717$

a. For $x = 28.5$: $z = (28.5 - 25)/4.87339717 = .72$

For $x = 29.5$: $z = (29.5 - 25)/4.87339717 = .92$

$P(28.5 \le x \le 29.5) = P(.72 \le z \le .92) = P(z \le .92) - P(z \le .72) = .8212 - .7642 = .0570$

b. For $x = 26.5$: $z = (26.5 - 25)/4.87339717 = .31$

$P(x \ge 26.5) = P(z \ge .31) = 1 - P(z \le .31) = 1 - .6217 = .3783$

c. For $x = 14.5$: $z = (14.5 - 25)/4.87339717 = -2.15$

For $x = 22.5$: $z = (22.5 - 25)/4.87339717 = -.51$

$P(14.5 \le x \le 22.5) = P(-2.15 \le z \le -.51) = P(z \le -.51) - P(z \le -2.15) = .3050 - .0158 = .2892$

6.79 $\mu = np = 100(.05) = 5$ and $\sigma = \sqrt{npq} = \sqrt{100(.05)(.95)} = 2.17944947$

a. For $x = 6.5$: $z = (6.5 - 5)/2.17944947 = .69$

$P(\text{shipment is accepted}) = P(x \le 6.5) = P(z \le .69) = .7549$

b. $P(\text{shipment is not accepted}) = 1 - P(\text{shipment is accepted}) = 1 - .7549 = .2451$

Supplementary Exercises

6.80 $\mu = 8$ minutes and $\sigma = 2$ minutes

a. For $x = 3$: $z = (3 - 8)/2 = -2.50$

$P(x < 3) = P(z < -2.50) = .0062$

b. For $x = 10$: $z = (10 - 8)/2 = 1.00$

For $x = 13$: $z = (13 - 8)/2 = 2.50$

$P(10 \le x \le 13) = P(1.00 \le z \le 2.50) = P(z \le 2.50) - P(z \le 1.00) = .9938 - .8413 = .1525$ or 15.25%

c. For $x = 6$: $z = (6 - 8)/2 = -1.00$

For $x = 12$: $z = (12 - 8)/2 = 2.00$

$P(6 \le x \le 12) = P(-1.00 \le z \le 2.00) = P(z \le 2.00) - P(z \le -1.00) = .9772 - .1587 = .8185$ or 81.85%

d. For $x = 16$: $z = (16 - 8)/2 = 4.00$

$P(x > 16) = P(z > 4.00) = 1 - P(z \le 4.00) = 1 - 1 = 0$ approximately

Although it is possible for a customer to wait for more than 16 minutes, the probability of this is very close to zero.

6.81 $\mu = \$87$ and $\sigma = \$22$

a. For $x = 114$: $z = (114 - 87)/22 = 1.23$

$P(x > 114) = P(z > 1.23) = 1 - P(z \le 1.23) = 1 - .8907 = .1093$

b. For $x = 40$: $z = (40 - 87)/22 = -2.14$

For $x = 60$: $z = (60 - 87)/22 = -1.23$

$P(40 < x < 60) = P(-2.14 < z < -1.23) = P(z \le -1.23) - P(z \le -2.14) = .1093 - .0162 = .0931$ or 9.31%

c. For $x = 70$: $z = (70 - 87)/22 = -.77$

For $x = 105$: $z = (105 - 87)/22 = .82$

$P(70 < x < 105) = P(-.77 < z < .82) = P(z \le .82) - P(z \le -.77) = .7939 - .2206 = .5733$ or 57.33%

d. For $x = 185$: $z = (185 - 87)/22 = 4.45$

$P(x > 185) = P(z > 4.45) = 1 - P(z \le 4.45) = 1 - 1 = 0$ approximately

Although it is possible for a customer to write a check for more than \$185, the probability of this is very close to zero.

6.82 $\mu = 16$ ounces and $\sigma = .18$ ounce

a. For $x = 16.20$: $z = (16.20 - 16)/.18 = 1.11$

For $x = 16.50$: $z = (16.50 - 16)/.18 = 2.78$

$P(16.20 \le x \le 16.50) = P(1.11 \le z \le 2.78) = P(z \le 2.78) - P(z \le 1.11) = .9973 - .8665 = .1308$

b. For $x = 15.70$: $z = (15.70 - 16)/.18 = -1.67$

$P(x < 15.70) = P(z < -1.67) = .0475$ or 4.75%

c. For $x = 15.20$: $z = (15.20 - 16)/.18 = -4.44$

$P(x < 15.20) = P(z < -4.44) = 0$ approximately

Although it is possible for a carton to contain less than 15.20 ounces, the probability of this is very close to zero.

6.83 $\mu = 50$ inches and $\sigma = .06$

For $x = 49.85$: $z = (49.85 - 50)/.06 = -2.50$

For $x = 50.15$: $z = (50.15 - 50)/.06 = 2.50$

$P(x < 49.85) + P(x > 50.15) = 1 - [P(49.85 \le x \le 50.15)] = 1 - [P(-2.50 \le z \le 2.50)]$

$= 1 - [P(z \le 2.50) - P(z \le -2.50)] = 1 - [.9938 - .0062] = 1 - .9876 = .0124$ or 1.24%

6.84 $\mu = 28$ minutes and $\sigma = 5$ minutes

Let x denote the morning commute time. We are to find x so that the area in the right tail of the normal distribution curve is .01. Thus, $z = 2.33$ and $x = \mu + z\sigma = 28 + (2.33)(5) = 39.65$ minutes. Thus, Jenn must leave by approximately 7:50 am, 40 minutes before she is due to arrive at work.

6.85 $\mu = 750$ hours and $\sigma = 50$ hours

a. The area in the right tail of the normal distribution curve is given to be .025, which gives $z = 1.96$. Then, $x = \mu + z\sigma = 750 + (1.96)(50) = 848$ hours

b. Area to the left of x is .80, which gives $z = .84$ approximately. Then, $x = \mu + z\sigma = 750 + (.84)(50) = 792$ hours

6.86 $\mu = 9.125$ inches and $\sigma = .06$ inch

For $x = 9$: $z = (9 - 9.125)/.06 = -2.08$

For $x = 9.25$: $z = (9.25 - 9.125)/.06 = 2.08$

$P(x < 9) + P(x > 9.25) = 1 - [P(9 \le x \le 9.25)] = 1 - [P(-2.08 \le z \le 2.08)]$

$= 1 - [P(z \le 2.08) - P(z \le -2.08)] = 1 - [.9812 - .0188] = 1 - .9624 = .0376$ or 3.76%

6.87 $\mu = np = 100(.80) = 80$ and $\sigma = \sqrt{npq} = \sqrt{100(.80)(.20)} = 4$

a. For $x = 84.5$: $z = (84.5 - 80)/4 = 1.13$

For $x = 85.5$: $z = (85.5 - 80)/4 = 1.38$

$P(84.5 \le x \le 85.5) = P(1.13 \le z \le 1.38) = P(z \le 1.38) - P(z \le 1.13) = .9162 - .8708 = .0454$

b. For $x = 74.5$: $z = (74.5 - 80)/4 = -1.38$

$P(x \le 74.5) = P(z \le -1.38) = .0838$

c. For $x = 74.5$: $z = (74.5 - 80)/4 = -1.38$

For $x = 87.5$: $z = (87.5 - 80)/4 = 1.88$

$P(74.5 \le x \le 87.5) = P(-1.38 \le z \le 1.88) = P(z \le 1.88) - P(z \le -1.38) = .9699 - .0838 = .8861$

d. For $x = 71.5$: $z = (71.5 - 80)/4 = -2.13$

For $x = 77.5$: $z = (77.5 - 80) / 4 = -.63$

$P(71.5 \le x \le 77.5) = P(-2.13 \le z \le -.63) = P(z \le -.63) - P(z \le -2.13) = .2643 - .0166 = .2477$

6.88 $\mu = np = 200(.80) = 160$ and $\sigma = \sqrt{npq} = \sqrt{200(.80)(.20)} = 5.65685425$

a. For $x = 149.5$: $z = (149.5 - 160)/5.65685425 = -1.86$

For $x = 150.5$: $z = (150.5 - 160)/5.65685425 = -1.68$

$P(149.5 \le x \le 150.5) = P(-1.86 \le z \le -1.68) = P(z \le -1.68) - P(z \le -1.86) = .0465 - .0314$
$= .0151$

b. For $x = 169.5$: $z = (169.5 - 160)/5.65685425 = 1.68$

$P(x \ge 169.5) = P(z \ge 1.68) = 1 - P(z \le 1.68) = 1 - .9535 = .0465$

c. For $x = 165.5$: $z = (165.5 - 160)/5.65685425 = .97$

$P(x \le 165.5) = P(z \le .97) = .8340$

d. For $x = 163.5$: $z = (163.5 - 160)/5.65685425 = .62$

For $x = 172.5$: $z = (172.5 - 160)/5.65685425 = 2.21$

$P(163.5 \le x \le 172.5) = P(.62 \le z \le 2.21) = P(z \le 2.21) - P(z \le .62) = .9864 - .7324 = .2540$

6.89 $\sigma = \$350$ and $P(x > 2500) = .15$

The area to the left of $x = 2500$ is $1 - .15 = .85$ and $z = 1.04$ approximately. Then, $\mu = x - z\sigma$ $= 2500 - (1.04)(350) = \$2136$. Thus, the mean monthly mortgage is approximately \$2136.

6.90 $\sigma = .18$ ounce and $P(x > 16) = .90$

The area to the left of $x = 16$ is $1 - .90 = .10$ and $z = -1.28$ approximately. Then, $\mu = x - z\sigma$ $= 16 - (-1.28)(.18) = 16.23$ ounces. Thus, the mean amount of ice cream put into these cartons by this machine should be approximately 16.23 ounces.

6.91 a. If $\$3500 = \$1000 + \frac{\$12}{\text{ft}} \cdot x$, where x is depth in feet, then $x = 208.33$. Hence, Company B charges more for depths of more than 208.33 ft.

$\mu = 250$ and $\sigma = 40$

For $x = 208.33$: $z = (208.33 - 250)/40 = -1.04$

$P(x > 208.33) = P(z > -1.04) = 1 - P(z \le -1.04) = 1 - .1492 = .8508$

The probability that Company B charges more than Company A to drill a well is .8508.

b. $\mu = 250$, so the mean amount charged by Company B is $\$1000 + \frac{\$12}{\text{ft}} \cdot 250 \text{ ft} = \$4000$.

6.92 $\mu = 290$ feet and $\sigma = 10$

The simplest solution to this exercise is obtained by using complementary events; i.e., P(at least one throw is 320 feet or longer) = 1 – P(all three throws are less than 320 feet). First, we find P(any one throw is less than 320 feet).

For $x = 320$: $z = (320 - 290)/10 = 3.00$

$P(x < 320) = P(z < 3.00) = .9987$

Since the three throws are independent,

P(all three throws are less than 320 feet) = $(.9987)^3 = .9961$.

Then, P(at least one throw is 320 feet or longer) = 1 – .9961 = .0039.

6.93 $\mu = 45{,}000$ and $\sigma = 2000$

First, we find the probability that one tire lasts at least 46,000 miles.

For $x = 46{,}000$: $z = (46{,}000 - 45{,}000)/2000 = .50$

$P(x \geq 46{,}000) = P(z \geq .50) = 1 - P(z \leq .50) = 1 - .6915 = .3085$

So, the probability of one tire lasting at least 46,000 miles is .3085. Then,

P(all four tires last more than 46,000 miles) = $(.3085)^4 = .0091$.

6.94 Plant A: $\mu = 20$ and $\sigma = 2$

For $x = 18$: $z = (18 - 20)/2 = -1.00$

For $x = 22$: $z = (22 - 20)/2 = 1.00$

$P(18 \leq x \leq 22) = P(-1.00 \leq z \leq 1.00) = P(z \leq 1.00) - P(z \leq -1.00) = .8413 - .1587 = .6826$

Plant B: $\mu = 19$ and $\sigma = 1$

For $x = 18$: $z = (18 - 19)/1 = -1.00$

For $x = 22$: $z = (22 - 19)/1 = 3.00$

$P(18 \leq x \leq 22) = P(-1.00 \leq z \leq 3.00) = P(z \leq 3.00) - P(z \leq -1.00) = .9987 - .1587 = .8400$

Plant B produces the greater proportion of pints that contain between 18% and 22% air.

6.95 $\mu = 0$ and $\sigma = 2$ mph

a. Let x be the error of these estimates in mph.

For $x = 5$: $z = (5 - 0)/2 = 2.50$

$P(x \geq 5) = P(z \geq 2.50) = 1 - P(z \leq 2.50) = 1 - .9938 = .0062$

b. We are given that the area to the left of x is .99, which gives z = 2.33 approximately. Then, $x = \mu + z\sigma = 0 + (2.33)(2) = 4.66$ mph ≈ 5 mph. So, the minimum estimate of speed at which a car should be cited for speeding is 60 + 5 = 65 mph.

6.96 $\mu = 45$ minutes and $\sigma = 3$ minutes

Let x be the amount of time Ashley spends commuting to work. We are given that the area to the left of x is .95, which gives z = 1.65 approximately. Then, $x = \mu + z\sigma = 45 + (1.65)(3) = 49.95 \approx 50$ minutes. Ashley should leave home at about 8:10 am in order to arrive at work by 9 am 95% of the time.

6.97 $\sigma = .07$ ounce and $P(x \geq 8) = .99$

The area to the left of $x = 8$ is $1 - .99 = .01$ and $z = -2.33$ approximately. Then, $\mu = x - z\sigma$ $= 8 - (-2.33)(.07) = 8.16$ ounces. Thus, the mean should be set at approximately 8.16 ounces.

6.98 $\mu = 500$ and $P(x < 430) = .20$

a. Then, $z = -.84$ approximately. Now $x = \mu + z\sigma$, so $\sigma = (x - \mu)/z = (430 - 500)/(-.84) = 83.33$.

b. For $x = 520$: $z = (520 - 500)/83.33 = .24$

$P(x > 520) = P(z > .24) = 1 - P(z \leq .24) = 1 - .5948 = .4052$ or 40.52%

6.99 Company A: $\mu = 8$ mm and $\sigma = .15$ mm

For $x = 7.8$: $z = (7.8 - 8)/.15 = -1.33$

For $x = 8.2$: $z = (8.2 - 8)/.15 = 1.33$

$P(7.8 \le x \le 8.2) = P(-1.33 \le z \le 1.33) = P(z \le 1.33) - P(z \le -1.33) = .9082 - .0918 = .8164$

Price per usable rod = 400/(.8164 × 10,000) ≈ \$0.048996

Company B: $\mu = 8$ mm and $\sigma = .12$ mm

For $x = 7.8$: $z = (7.8 - 8) / .12 = -1.67$

For $x = 8.2$: $z = (8.2 - 8) / .12 = 1.67$

$P(7.8 \le x \le 8.2) = P(-1.67 \le z \le 1.67) = P(z \le 1.67) - P(z \le -1.67) = .9525 - .0475 = .9050$

Price per usable rod = 460/(.9050 x 10,000) ≈ \$0.050829

Hence, the Alpha Corporation should choose Company A as a supplier.

6.100 a. Answers will vary.

b. Single-number bet: $np = (25)\left(\frac{1}{38}\right) = .658 < 5$, so we cannot use the standard normal distribution to find the probability. The gambler comes out ahead if his number comes up at least once:

P (at least one success) = 1 – P(all losses) = $1 - \left(1 - \frac{1}{38}\right)^{25} = .4866$

Color bet: $np = (25)\left(\frac{18}{38}\right) = 11.84$, so we can use the normal distribution to approximate the probability.

$\mu = np = 25\left(\frac{18}{38}\right) = 11.84$ and $\sigma = \sqrt{npq} = \sqrt{25\left(\frac{18}{38}\right)\left(\frac{20}{38}\right)} = 2.49653500$

The gambler will come out ahead if he wins 13 bets or more.

For $x = 12.5$: $z = (12.5 - 11.84)/2.49653500 = .26$

$P(x \ge 12.5) = P(z \ge .26) = 1 - P(z \le .26) = 1 - .6026 = .3974$

The gambler has a better chance of coming out ahead with the single-number bet.

6.101 a. Let x = number of ticket holders who show up.

Then, x is a binomial random variable with $n = 65$ and $p = 1 - .10 = .90$

$\mu = np = 65(.90) = 58.5$ and $\sigma = \sqrt{npq} = \sqrt{65(.90)(.10)} = 2.41867732$

Using a normal approximation with correction for continuity:

For $x = 60.5$: $z = (60.5 - 58.5)/2.41867732 = .83$

$P(x \le 60.5) = P(z \le .83) = .7967$

b. Let n = number of tickets sold

Since μ and σ both depend on n, it is not easy to solve for n directly. Instead, use trial and error.

In part a, $n = 65$ was too large, since $P(x \le 60.5) = P(z \le .83) = .7967 < .95$.

Try $n = 62$: $\mu = np = 62(.90) = 55.8$ and $\sigma = \sqrt{npq} = \sqrt{62(.90)(.10)} = 2.36220236$

For $x = 60.5$: $z = (60.5 - 55.8)/2.36220236 = 1.99$

$P(x \le 60.5) = P(z \le 1.99) = .9767 > .95$.

Thus, $n = 62$ satisfies the requirement.

To see if n may be increased, try $n = 63$.

$\mu = np = 63(.90) = 56.7$ and $\sigma = \sqrt{npq} = \sqrt{63(.90)(.10)} = 2.38117618$

For $x = 60.5$: $z = (60.5 - 56.7)/2.38117618 = 1.60$

$P(x \le 60.5) = P(z \le 1.60) = .9452 < .95$

Thus, $n = 63$ is too large.

Therefore, the largest number of tickets the company can sell and be at least 95% sure that the bus can hold all ticket holders who show up is 62.

6.102 a. $\mu = 2$ minutes and $\sigma = .5$ minute

First, we find the probability that it will take less than 1 minute for one customer to be served.

For $x = 1$: $z = (1 - 2)/.5 = -2.00$

$P(x < 1) = P(z < -2.00) = .0228$

Since customers are independent,

P(two customers take less than one minute each) = (.0228)(.0228) = .0005

b. Using complementary events,

P(at least one of the four needs 2.25 minutes) = 1 – P(each of the four needs 2.25 minutes or less), so we first find the probability that one customer needs 2.25 minutes or less to be served.

For $x = 2.25$: $z = (2.25 - 2)/.5 = .50$

$P(x \le 2.25) = P(z \le .5) = .6915$

Since customers are independent,

P(at least one of the four needs 2.25 minutes) = 1 – P(each of the four needs 2.25 minutes or less) $= 1 - (.6915)^4 = .7714$.

6.103 $\mu = np = 15(.02) = .30$ and $\sigma = \sqrt{npq} = \sqrt{15(.02)(.98)} = .54221767$

Since $np < 5$, the normal approximation to the binomial is not appropriate. The Empirical Rule requires a bell-shaped distribution, and this distribution is skewed right. By the Empirical Rule, approximately 68% of the observations fall in the interval $\mu \pm \sigma$, approximately 95% fall in the interval $\mu \pm 2\sigma$, and about 99.7% fall in the interval $\mu \pm 3\sigma$. These intervals are −.24 to .84, −.78 to 1.38, and −1.33 to 1.93, respectively. Using the normal approximation with continuity correction,

For $x = -.74$: $z = (-.74 - .3)/.54221767 = -1.92$

For $x = 1.34$: $z = (1.34 - .3)/.54221767 = 1.92$

$P(-.74 \le x \le 1.34) = P(-1.92 \le z \le 1.92) = P(z \le 1.92) - P(z \le -1.92) = .9726 - .0274 = .9452 > .68$

For $x = -1.28$: $z = (-1.28 - .3)/.54221767 = -2.91$

For $x = 1.88$ $z = (1.88 - .3)/.54221767 = 2.91$

$P(-1.28 \le x \le 1.88) = P(-2.91 \le z \le 2.91) = P(z \le 2.91) - P(z \le -2.91) = .9982 - .0018 = .9964 > .95$

For $x = -1.83$: $z = (-1.83 - .3)/.54221767 = -3.93$

For $x = 2.43$: $z = (2.43 - .3)/.54221767 = 3.93$

$P(-1.83 \le x \le 2.43) = P(-3.93 \le z \le 3.93) = P(z \le 3.93) - P(z \le -3.93) = 1 - 0 \approx 1 > .997$

6.104 a. $P(.3 \le z \le .4) = P(z \le .4) - P(z \le .3) = .6554 - .6179 = .0375$

b. $P(-.1 \le z \le .4) = P(z \le .4) - P(z \le -.1) = .6554 - .4602 = .1952$

c. P(at least one out of five games, the ball lands in the required slot) = $1 - P$(none)

For one game, the probability the ball lands outside the required slot is $1 - P(-.1 \le z \le .4)$ $= 1 - .1952 = .8048$. Since the games are independent,

$1 - P(\text{none}) = 1 - (.8048)^5 = 1 - .3376 = .6624$

d. Note that 100 out of 500 games is equivalent to 1 out of 5 games. Then, the probability that the ball lands outside the required slot for one game is

$1 - P(.4 \le z \le .5) = 1 - [P(z \le .5) - P(z \le .4)] = 1 - [.6915 - .6554] = 1 - .0361 = .9639$

Since the games are independent, $1 - P(\text{none}) = 1 - (.9639)^5 = 1 - .8321 = .1679$

6.105 $\mu = 8$ and $P(x > 8.2) = .03$. Then, $z = 1.88$ approximately. Now $x = \mu + z\sigma$, so

$\sigma = (x - \mu)/z = (8.2 - 8)/(1.88) = .106$.

Self-Review Test

1. a **2.** a **3.** d **4.** b **5.** a **6.** c **7.** b **8.** b

9. a. $P(.85 \le z \le 2.33) = P(z \le 2.33) - P(z \le .85) = .9901 - .8023 = .1878$

b. $P(-2.97 \le z \le 1.49) = P(z \le 1.49) - P(z \le -2.97) = .9319 - .0015 = .9304$

c. $P(z \le -1.29) = .0985$

d. $P(z > -.74) = 1 - P(z \le -.74) = 1 - .2296 = .7704$

10. a. $z = -1.28$ approximately b. $z = .61$

c. $z = 1.65$ approximately d. $z = -1.07$ approximately

11. $\mu = 45{,}000$ miles and $\sigma = 2360$ miles

a. For $x = 42{,}000$: $z = (42{,}000 - 45{,}000)/2360 = -1.27$

For $x = 46{,}000$: $z = (46{,}000 - 45{,}000)/2360 = .42$

$P(42{,}000 < x < 45{,}000) = P(-1.27 < z < .42) = P(z < .42) - P(z < -1.27) = .6628 - .1020 = .5608$

b. For $x = 38{,}000$ $z = (38{,}000 - 45{,}000)/2360 = -2.97$

$P(x < 38{,}000) = P(z < -2.97) = .0015$

c. For $x = 50{,}000$: $z = (50{,}000 - 45{,}000)/2360 = 2.12$

$P(x > 50{,}000) = P(z > 2.12) = 1 - P(z \le 2.12) = 1 - .9830 = .0170$

d. For $x = 46{,}500$: $z = (46{,}500 - 45{,}000)/2360 = .64$

For $x = 47{,}500$: $z = (47{,}500 - 45{,}000)/2360 = 1.06$

$P(46{,}500 < x < 47{,}500) = P(.64 < z < 1.06) = P(z < 1.06) - P(z < .64) = .8554 - .7389 = .1165$

12. $\mu = 45{,}000$ miles and $\sigma = 2360$ miles

a. For .06 area in the right tail of the normal distribution curve, $z \approx 1.55$. Then,

$x = \mu + z\sigma = 45{,}000 + (1.55)(2360) = 48{,}658$ miles

b. For .02 area in the left tail of the normal distribution curve, $z \approx -2.05$. Then,

$x = \mu + z\sigma = 45{,}000 + (-2.05)(2360) = 40{,}162$ miles

13. $\mu = np = 800(.15) = 120$ and $\sigma = \sqrt{npq} = \sqrt{800(.15)(.85)} = 10.09950494$

a. i. For $x = 114.5$: $z = (114.5 - 120)/10.09950494 = -.54$

For $x = 115.5$: $z = (115.5 - 120)/10.09950494 = -.45$

$P(114.5 \le x \le 115.5) = P(-.54 \le z \le -.45) = P(z \le -.45) - P(z \le -.54) = .3264 - .2946 = .0318$

ii. For $x = 102.5$: $z = (102.5 - 120)/10.09950494 = -1.73$

For $x = 142.5$: $z = (142.5 - 120)/10.09950494 = 2.23$

$P(102.5 \le x \le 142.5) = P(-1.73 \le z \le 2.23) = P(z \le 2.23) - P(z \le -1.73) = .9871 - .0418$
$= .9453$

iii. For $x = 106.5$: $z = (106.5 - 120)/10.09950494 = -1.34$

$P(x \ge 106.5) = P(z \ge -1.34) = 1 - P(z \le -1.34) = 1 - .0901 = .9099$

iv. For $x = 100.5$: $z = (100.5 - 120)/10.09950494 = -1.93$

$P(x \le 100.5) = P(z \le -1.93) = .0268$

v. For $x = 110.5$: $z = (110.5 - 120)/10.09950494 = -.94$

For $x = 123.5$: $z = (123.5 - 120)/10.09950494 = .35$

$P(110.5 \le x \le 123.5) = P(-.94 \le z \le .35) = P(z \le .35) - P(z \le -.94) = .6368 - .1736 = .4632$

b. P(at least 675 do not have wheat intolerance) = P(at most 125 have wheat intolerance) = $P(x \le 125)$

For $x = 125.5$: $z = (125.5 - 120)/10.09950494 = .54$

$P(x \le 125.5) = P(z \le .54) = .7054$

c. P(between 682 and 697 do not have wheat intolerance) =

P(between 103 and 118 have wheat intolerance) = $P(102.5 \le x \le 118.5)$

For $x = 102.5$: $z = (102.5 - 120)/10.09950494 = -1.73$

For $x = 118.5$: $z = (118.5 - 120)/10.09950494 = -.15$

$P(102.5 \le x \le 118.5) = P(-1.73 \le z \le -.15) = P(z \le -.15) - P(z \le -1.73) = .4404 - .0418 = .3986$