Chapter Five

Section 5.1

5.1 A variable whose value is determined by the outcome of a random experiment is called a **random variable**. A random variable that assumes countable values is called a **discrete random variable**. The number of cars owned by a randomly selected individual is an example of a discrete random variable. A random variable that can assume any value contained in one or more intervals is called a **continuous random variable**. An example of a continuous random variable is the amount of time taken by a randomly selected student to complete a statistics exam.

5.2

a. continuous	b. discrete
c. discrete	d. continuous
e. discrete	f. continuous

5.3

a. discrete	b. continuous
c. continuous	d. discrete
e. discrete	f. continuous

5.4 The number of households x watching ABC is a discrete random variable because the values of x are countable: 0, 1, 2, 3, 4 and 5.

5.5 The number of cars x that stop at the Texaco station is a discrete random variable because the values of x are countable: 0, 1, 2, 3, 4, 5 and 6.

Section 5.2

5.6 The **probability distribution** of a discrete random variable lists all the possible values that the random variable can assume and their corresponding probabilities. Table 5.3 in Example 5-1 in the text displays a probability distribution for a discrete random variable. The probability distribution of a discrete random variable can be presented in the form of a mathematical formula, a table, or graph.

5.7 1. The probability assigned to each value of a random variable x lies in the range 0 to 1; that is, $0 \le P(x) \le 1$ for each x.

2. The sum of the probabilities assigned to all possible values of x is equal to 1; that is, $\sum P(x) = 1$.

5.8 a. This table does represent a valid probability distribution of x because it satisfies both conditions required for a valid probability distribution.

b. This table does not represent a valid probability distribution of x because the sum of the probabilities of all outcomes listed in the table is not 1, which violates the second condition of a probability distribution.

c. This table does not represent a valid probability distribution of x because the probability of $x = 7$ is negative, which violates the first condition of a probability distribution.

5.9 a. This table does not satisfy the first condition of a probability distribution because the probability of $x = 5$ is negative. Hence, it does not represent a valid probability distribution of x.

b. This table represents a valid probability distribution of x because it satisfies both conditions required for a valid probability distribution.

c. This table does not represent a valid probability distribution of x because the sum of the probabilities of all outcomes listed in the table is not 1, which violates the second condition of a probability distribution.

5.10 a. $P(x = 3) = .15$

b. $P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2) = .11 + .19 + .28 = .58$

c. $P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6) = .12 + .09 + .06 = .27$

d. $P(1 \leq x \leq 4) = P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) = .19 + .28 + .15 + .12 = .74$

e. $P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) = .11 + .19 + .28 + .15 = .73$

f. $P(x > 2) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) = .15 + .12 + .09 + .06 = .42$

g. $P(2 \leq x \leq 5) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) = .28 + .15 + .12 + .09 = .64$

5.11 a. $P(x = 1) = .17$

b. $P(x \leq 1) = P(x = 0) + P(x = 1) = .03 + .17 + = .20$

c. $P(x \geq 3) = P(x = 3) + P(x = 4) + P(x = 5) = .31 + .15 + .12 = .58$

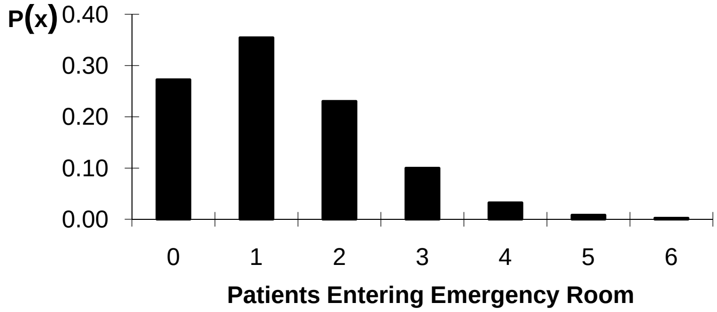
d. $P(0 \leq x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2) = .03 + .17 + .22 = .42$

e. $P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2) = .03 + .17 + .22 = .42$

f. $P(x > 3) = P(x = 4) + P(x = 5) = .15 + .12 = .27$

g. $P(2 \leq x \leq 4) = P(x = 2) + P(x = 3) + P(x = 4) = .22 + .31 + .15 = .68$

5.12 a. Let x = number of patients entering the emergency room during a one-hour period.



b. i. P(2 or more) = $P(x \geq 2) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)$

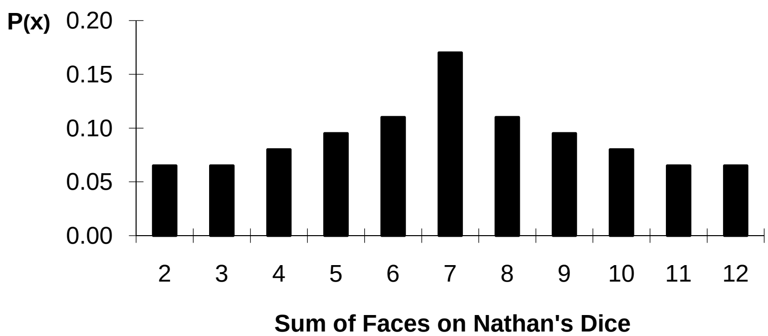
$= .2303 + .0998 + .0324 + .0084 + .0023 = .3732$

ii. P(exactly 5) = $P(x = 5) = .0084$

iii. P(fewer than 3) = $P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2) = .2725 + .3543 + .2303 = .8571$

iv. P(at most 1) = $P(x \leq 1) = P(x = 0) + P(x = 1) = .2725 + .3543 = .6268$

5.13 a.



b. i. P(an even number) = $P(x = 2) + P(x = 4) + P(x = 6) + P(x = 8) + P(x = 10) + P(x = 12)$

$= .065 + .08 + .11 + .11 + .08 + .065 = .51$

ii. P(7 or 11) = $P(x = 7) + P(x = 11) = .17 + .065 = .235$

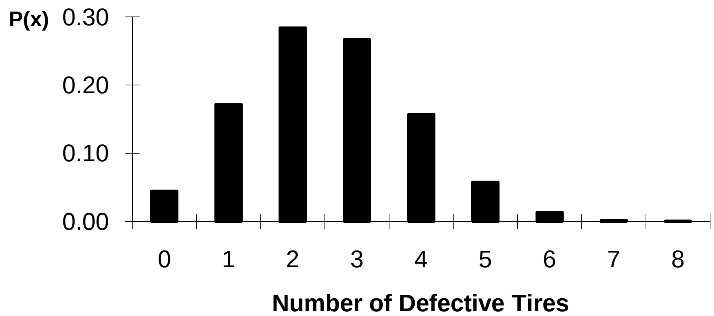
iii. P(4 to 6) = $P(4 \leq x \leq 6) = P(x = 4) + P(x = 5) + P(x = 6) = .08 + .095 + .11 = .285$

iv. P(no less than 9) = $P(x \geq 9) = P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12)$

$= .095 + .08 + .065 + .065 = .305$

5.14 a. Let x denote the number of defective tires on an H2 limo.

x	$P(x)$
0	59/1300 = .045
1	224/1300 = .172
2	369/1300 = .284
3	347/1300 = .267
4	204/1300 = .157
5	76/1300 = .058
6	18/1300 = .014
7	2/1300 = .002
8	1/1300 = .001



b. The probabilities listed in the table of part a are exact because they are based on data from the entire population (fleet of 1300 limos).

c. i. $P(x = 0) = .045$

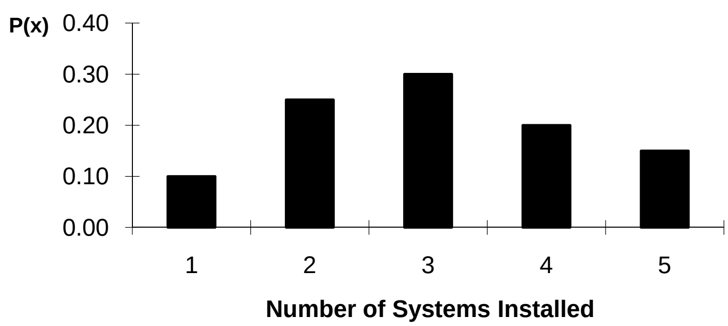
ii. $P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) = .045 + .172 + .284 + .267 = .768$

iii. $P(3 \leq x < 7) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) = .267 + .157 + .058 + .014 = .496$

iv. $P(x \geq 2) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7) + P(x = 8)$
$= .284 + .267 + .157 + .058 + .014 + .002 + .001 = .783$

5.15 a.

x	$P(x)$
1	8/80 = .10
2	20/80 = .25
3	24/80 = .30
4	16/80 = .20
5	12/80 = .15



b. The probabilities listed in the table of part a are approximate because they are obtained from a sample of 80 days.

c. i. $P(x = 3) = .30$

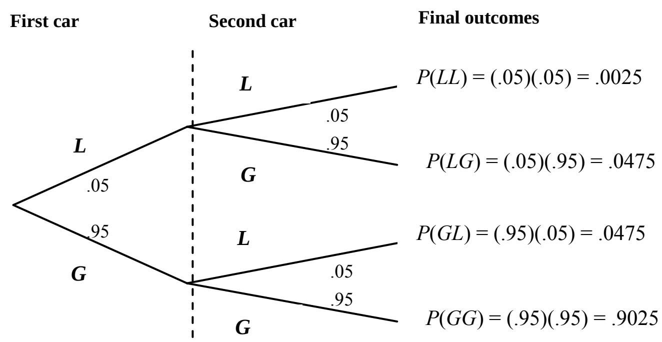
ii. $P(x \geq 3) = P(x = 3) + P(x = 4) + P(x = 5) = .30 + .20 + .15 = .65$

iii. $P(2 \leq x \leq 4) = P(x = 2) + P(x = 3) + P(x = 4) = .25 + .30 + .20 = .75$

iv. $P(x < 4) = P(x = 1) + P(x = 2) + P(x = 3) = .10 + .25 + .30 = .65$

5.16 Let L = car selected is a lemon and G = car selected is not a lemon.

Then, $P(L) = .05$ and $P(G) = 1 - .05 = .95$. Note that the first and second events are independent.



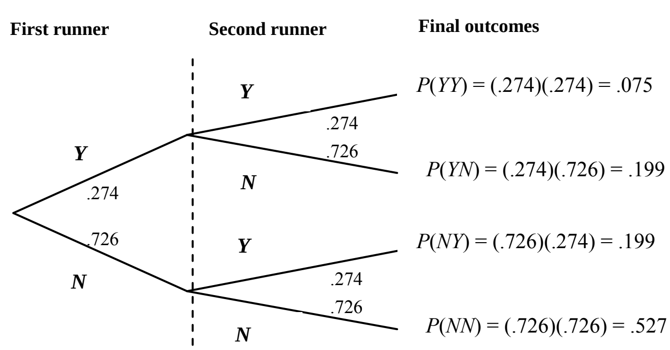
$P(x = 0) = P(GG) = .9025$, $P(x = 1) = P(LG) + P(GL) = .0475 + .0475 = .0950$,

$P(x = 2) = P(LL) = .0025$

x	$P(x)$
0	.9025
1	.0950
2	.0025

5.17 Let Y = runner selected finished the race in 49:42 or less and N = runner selected did not finish the race in 49:42 or less.

Then $P(Y) = .274$ and $P(N) = 1 - .274 = .726$. Note that the first and second events are independent.

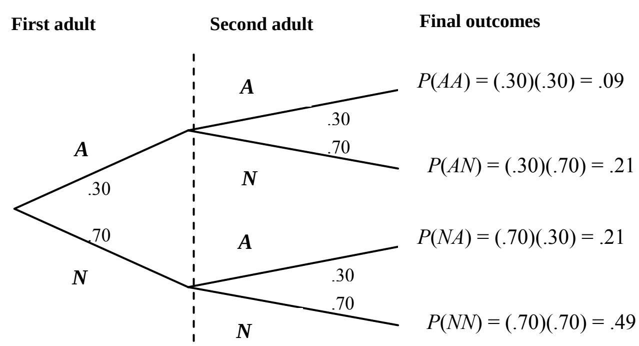


$P(x = 0) = P(NN) = .527$, $P(x = 1) = P(YN) + P(NY) = .199 + .199 = .398$, $P(x = 2) = P(YY) = .075$

x	$P(x)$
0	.527
1	.398
2	.075

5.18 Let A = adult selected is against using animals for research and N = adult selected is not against using animals for research.

Then, $P(A) = .30$ and $P(N) = 1 - .30 = .70$. Note that the first and second events are independent.

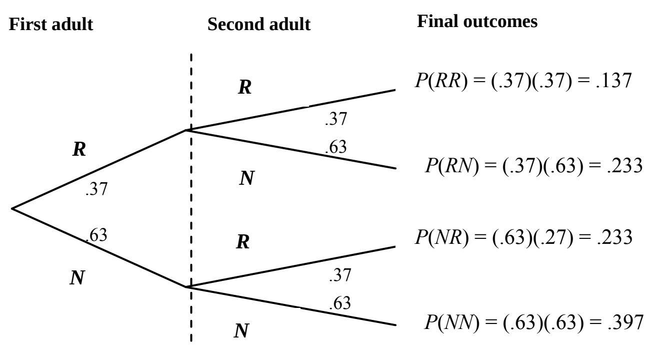


$P(x = 0) = P(NN) = .49$, $P(x = 1) = P(AN) + P(NA) = .21 + .21 = .42$, $P(x = 2) = P(AA) = .09$

x	$P(x)$
0	.49
1	.42
2	.09

5.19 Let R = adult encounters "rude and disrespectful behavior" often and N = adult does not encounter "rude and disrespectful behavior" often.

Then $P(R) = .37$ and $P(N) = 1 - .37 = .63$. Note that the first and second events are independent.

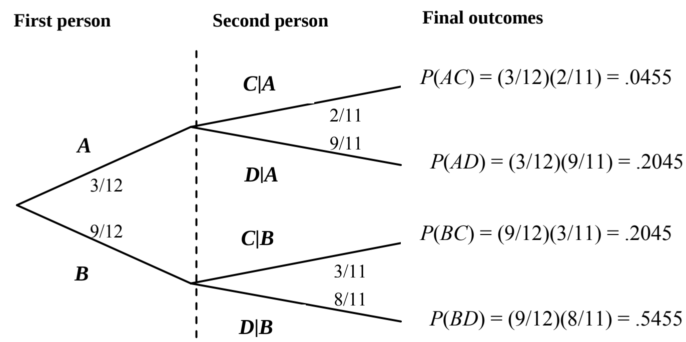


$P(x = 0) = P(NN) = .397$, $P(x = 1) = P(RN) + P(NR) = .233 + .233 = .466$,

$P(x = 2) = P(RR) = .137$

x	$P(x)$
0	.397
1	.466
2	.137

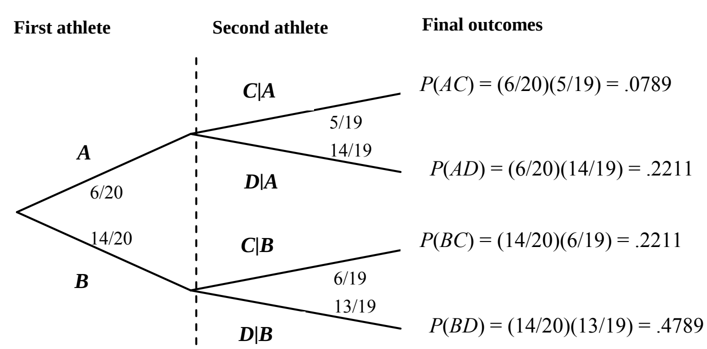
5.20 Let A = first person selected is left-handed, B = first person selected is right-handed, C = second person selected is left-handed, and D = second person selected is right-handed.



$P(x = 0) = P(BD) = .5455$, $P(x =1) = P(AD) + P(BC) = .2045 + .2045 = .4090$,
$P(x = 2) = P(AC) = .0455$

x	$P(x)$
0	.5455
1	.4090
2	.0455

5.21 Let A = first athlete selected used drugs, B = first athlete selected did not use drugs, C = second athlete selected used drugs, and D = second athlete selected did not use drugs.



$P(x = 0) = P(BD) = .4789$, $P(x =1) = P(AD) + P(BC) = .2211 + .2211 = .4422$,
$P(x = 2) = P(AC) = .0789$

x	$P(x)$
0	.4789
1	.4422
2	.0789

Sections 5.3 - 5.4

5.22 The **mean** of discrete random variable x is the value that is expected to occur per repetition, on average, if an experiment is repeated a large number of times. It is denoted by μ and calculated as $\mu = \sum xP(x)$. The **standard deviation** of a discrete random variable x measures the spread of its probability distribution. It is denoted by σ and is calculated as $\sigma = \sqrt{\sum x^2 P(x) - \mu^2}$.

5.23 a.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.16	.00	0	.00
1	.27	.27	1	.27
2	.39	.78	4	1.56
3	.18	.54	9	1.62
		$\sum xP(x) = 1.59$		$\sum x^2P(x) = 3.45$

$\mu = \sum xP(x) = 1.590$

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2}\,) = \sqrt{3.45 - (1.59)^2} = .960$

b.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
6	.40	2.40	36	14.40
7	.26	1.82	49	12.74
8	.21	1.68	64	13.44
9	.13	1.17	81	10.53
		$\sum xP(x) = 7.07$		$\sum x^2P(x) = 51.11$

$\mu = \sum xP(x) = 7.070$

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{51.11 - (7.07)^2} = 1.061$

5.24 a.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
3	.09	.27	9	.81
4	.21	.84	16	3.36
5	.34	1.70	25	8.50
6	.23	1.38	36	8.28
7	.13	.91	49	6.37
		$\sum xP(x) = 5.10$		$\sum x^2P(x) = 27.32$

$\mu = \sum xP(x) = 5.10$

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{27.32 - (5.10)^2} = 1.145$

b.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.43	.00	0	.00
1	.31	.31	1	.31
2	.17	.34	4	.68
3	.09	.27	9	.81
		$\sum xP(x) = .92$		$\sum x^2P(x) = 1.8$

$\mu = \sum xP(x) = .92$

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{1.80 - (.92)^2} = .977$

5.25

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.73	.00	0	.00
1	.16	.16	1	.16
2	.06	.12	4	.24
3	.04	.12	9	.36
4	.01	.04	16	.16
		$\sum xP(x) = .44$		$\sum x^2P(x) = .92$

$\mu = \sum xP(x) = .440$ errors

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{.92 - (.44)^2} = .852$ errors

5.26

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.36	.00	0	.00
1	.24	.24	1	.24
2	.18	.36	4	.72
3	.10	.30	9	.90
4	.07	.28	16	1.12
5	.05	.25	25	1.25
		$\sum xP(x) = 1.43$		$\sum x^2P(x) = 4.23$

$\mu = \sum xP(x) = 1.43$ magazines

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{4.23 - (1.43)^2} = 1.478$ magazines

5.27 Let x be the number of camcorders sold on a given day at an electronics store.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.05	.00	0	.00
1	.12	.12	1	.12
2	.19	.38	4	.76
3	.30	.90	9	2.70
4	.20	.80	16	3.20
5	.10	.50	25	2.50
6	.04	.24	36	1.44
		$\sum xP(x) = 2.94$		$\sum x^2P(x) = 10.72$

$\mu = \sum xP(x) = 2.94$ camcorders

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{10.72 - (2.94)^2} = 1.441$ camcorders

On average, 2.94 camcorders are sold per day at this store.

5.28 Let x be the number of patients entering the emergency room during a one-hour period at Millard Fellmore Memorial Hospital.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.2725	.0000	0	.0000
1	.3543	.3543	1	.3543
2	.2303	.4606	4	.9212
3	.0998	.2994	9	.8982
4	.0324	.1296	16	.5184
5	.0084	.0420	25	.2100
6	.0023	.0138	36	.0828
		$\sum xP(x) = 1.2997$		$\sum x^2P(x) = 2.9849$

$\mu = \sum xP(x) = 1.2997$ patients

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{2.9849 - (1.2997)^2} = 1.1383$ patients

5.29

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.25	.00	0	.00
1	.50	.50	1	.50
2	.25	.50	4	1.00
		$\sum xP(x) = 1.00$		$\sum x^2P(x) = 1.50$

$\mu = \sum xP(x) = 1.00$ head

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{1.50 - (1.00)^2} = .707$ head

On average, we will obtain 1 head in every two tosses of the coin.

5.30

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.14	.00	0	.00
1	.28	.28	1	.28
2	.22	.44	4	.88
3	.18	.54	9	1.62
4	.12	.48	16	1.92
5	.06	.30	25	1.50
		$\sum xP(x) = 2.04$		$\sum x^2P(x) = 6.20$

$\mu = \sum xP(x) = 2.04$ potential weapons

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{6.20 - (2.04)^2} = 1.428$ potential weapons

On average, 2.04 potential weapons are found per day.

5.31 Let x be the number of defective tires on an H2 limo.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.045	.000	0	.000
1	.172	.172	1	.172
2	.284	.568	4	1.136
3	.267	.801	9	2.403
4	.157	.628	16	2.512
5	.058	.290	25	1.450
6	.014	.084	36	.504
7	.002	.014	49	.098
8	.001	.008	64	.064
		$\sum xP(x) = 2.565$		$\sum x^2P(x) = 8.339$

$\mu = \sum xP(x) = 2.565$ defective tires

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{8.339 - (2.565)^2} = 1.327$ defective tires

There is an average of 2.565 defective tires per limo, with a standard deviation of 1.327 tires.

5.32 Let x be the number of remote starting systems installed on a given day at Al's.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
1	.10	.10	1	.10
2	.25	.50	4	1.00
3	.30	.90	9	2.70
4	.20	.80	16	3.20
5	.15	.75	25	3.75
		$\sum xP(x) = 3.05$		$\sum x^2P(x) = 10.75$

$\mu = \sum xP(x) = 3.05$ installations

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{10.75 - (3.05)^2} = 1.203$ installations

The average number of installations is 3.05 per day with a standard deviation of 1.203 installations.

5.33

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.9025	.0000	0	.0000
1	.0950	.0950	1	.0950
2	.0025	.0050	4	.0100
		$\sum xP(x) = .100$		$\sum x^2P(x) = .105$

$\mu = \sum xP(x) = .10$ lemon

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{.105 - (.100)^2} = .308$ lemon

5.34

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.527	.000	0	.000
1	.398	.398	1	.398
2	.075	.150	4	.300
		$\sum xP(x) = .548$		$\sum x^2P(x) = .698$

$\mu = \sum xP(x) = .548$ runner

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{.698 - (.548)^2} = .631$ runner

5.35

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.10	.00	0	.00
2	.45	.90	4	1.80
5	.30	1.50	25	7.50
10	.15	1.50	100	15.00
		$\sum xP(x) = 3.9$		$\sum x^2P(x) = 24.30$

$\mu = \sum xP(x) = \$3.9$ million

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{24.30 - (3.9)^2} = \$3.015$ million

The contractor is expected to make an average of $3.9 million profit with a standard deviation of $3.015 million.

5.36 Note that the price of the ticket ($2) must be deducted from the amount won. For example, the $5 prize results in a net gain of $5 – $2 = $3.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
−2	.8894	−1.7788	4	3.5576
3	.1000	.3000	9	.9000
8	.0100	.0800	64	.6400
998	.0005	.4990	996,004	498.0020
4998	.0001	.4998	24,980,004	2498.0004
		$\sum xP(x) = -.40$		$\sum x^2P(x) = 3001.10$

$\mu = \sum xP(x) = -\$0.40$

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{3001.10 - (-.40)^2} = \$54.78$

On average, the players who play this game are expected to lose $0.40 per ticket with a standard deviation of $54.78.

5.37

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.5455	.0000	0	.0000
1	.4090	.4090	1	.4090
2	.0455	.0910	4	.1820
		$\sum xP(x) = .500$		$\sum x^2P(x) = .591$

$\mu = \sum xP(x) = .500$ person

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{.591 - (.500)^2} = .584$ person

5.38

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.4789	.0000	0	.0000
1	.4422	.4422	1	.4422
2	.0789	.1578	4	.3156
		$\sum xP(x) = .600$		$\sum x^2P(x) = .7578$

$\mu = \sum xP(x) = .600$ athlete

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{.7578 - (.600)^2} = .631$ athlete

Section 5.5

5.39 $3! = 3 \cdot 2 \cdot 1 = 6$

$(9 - 3)! = 6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880$

$(14 - 12)! = 2! = 2 \cdot 1 = 2$

$${}_5C_3 = \frac{5!}{3!(5-3)!} = \frac{5!}{3!\,2!} = \frac{120}{(6)(2)} = 10 \qquad {}_7C_4 = \frac{7!}{4!(7-4)!} = \frac{7!}{4!\,3!} = \frac{5040}{(24)(6)} = 35$$

$${}_9C_3 = \frac{9!}{3!(9-3)!} = \frac{9!}{3!\,6!} = \frac{362{,}880}{(6)(720)} = 84 \qquad {}_4C_0 = \frac{4!}{0!(4-0)!} = \frac{4!}{0!\,4!} = \frac{24}{(1)(24)} = 1$$

$${}_3C_3 = \frac{3!}{3!(3-3)!} = \frac{3!}{3!\,0!} = \frac{6}{(6)(1)} = 1 \qquad {}_6P_2 = \frac{6!}{(6-2)!} = \frac{6!}{4!} = \frac{720}{24} = 30$$

$${}_8P_4 = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{40{,}320}{24} = 1680$$

5.40 $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

$11! = 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 39{,}916{,}800$

$(7 - 2)! = 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$(15 - 5)! = 10! = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3{,}628{,}800$

$${}_8C_2 = \frac{8!}{2!(8-2)!} = \frac{8!}{2!\,6!} = \frac{40{,}320}{(2)(720)} = 28 \qquad {}_5C_0 = \frac{5!}{0!(5-0)!} = \frac{5!}{0!\,5!} = \frac{120}{(1)(120)} = 1$$

$${}_5C_5 = \frac{5!}{5!(5-5)!} = \frac{5!}{5!\,0!} = \frac{120}{(120)(1)} = 1 \qquad {}_6C_4 = \frac{6!}{4!(6-4)!} = \frac{6!}{4!\,2!} = \frac{720}{(24)(2)} = 15$$

$${}_{11}C_7 = \frac{11!}{7!(11-7)!} = \frac{11!}{7!\,4!} = \frac{39{,}916{,}800}{(5040)(24)} = 330 \qquad {}_9P_6 = \frac{9!}{(9-6)!} = \frac{9!}{3!} = \frac{362{,}880}{6} = 60{,}480$$

$${}_{12}P_8 = \frac{12!}{(12-8)!} = \frac{12!}{4!} = \frac{479{,}001{,}600}{24} = 19{,}958{,}400$$

5.41 $_9C_2 = \frac{9!}{2!(9-2)!} = \frac{9!}{2!\,7!} = \frac{362{,}880}{(2)(5040)} = 36$; $_9P_2 = \frac{9!}{(9-2)!} = \frac{9!}{7!} = \frac{362{,}880}{5040} = 72$

5.42 $_{25}C_2 = \frac{25!}{2!(25-2)!} = \frac{25!}{2!\,23!} = 300$; $_{25}P_2 = \frac{25!}{(25-2)!} = \frac{25!}{23!} = 600$

5.43 $_{12}C_3 = \frac{12!}{3!(12-3)!} = \frac{12!}{3!\,9!} = 220$; $_{12}P_3 = \frac{12!}{(12-3)!} = \frac{12!}{9!} = 1320$

5.44 $_{25}C_4 = \frac{25!}{4!(25-4)!} = \frac{25!}{4!\,21!} = 12{,}650$; $_{25}P_4 = \frac{25!}{(25-4)!} = \frac{25!}{21!} = 303{,}600$

5.45 $_{20}C_6 = \frac{20!}{6!(20-6)!} = \frac{20!}{6!\,14!} = 38{,}760$; $_{20}P_6 = \frac{20!}{(20-6)!} = \frac{20!}{14!} = 27{,}907{,}200$

5.46 $_{16}C_2 = \frac{16!}{2!(16-2)!} = \frac{16!}{2!\,14!} = 120$; $_{16}P_2 = \frac{16!}{(16-2)!} = \frac{16!}{14!} = 240$

5.47 $_{20}C_9 = \frac{20!}{9!(20-9)!} = \frac{20!}{9!\,11!} = 167{,}960$

5.48 $_{15}C_5 = \frac{15!}{5!(15-5)!} = \frac{15!}{5!\,10!} = 3003$

Section 5.6

5.49 a. An experiment that satisfies the following four conditions is called a **binomial experiment**:

i. There are n identical trials. In other words, the given experiment is repeated n times, where n is a positive integer. All these repetitions are performed under identical conditions.

ii. Each trial has two and only two outcomes. These outcomes are usually called a *success* and a *failure*.

iii. The probability of success is denoted by p and that of failure by q, and $p + q = 1$. The probability of p and q remain constant for each trial.

iv. The trials are independent. In other words, the outcome of one trial does not affect the outcome of another trial.

b. Each repetition of a binomial experiment is called a **trial**.

c. A **binomial random variable** x represents the number of successes in n independent trials of a binomial experiment.

5.50 The parameters of the binomial distribution are n and p, which stand for the total number of trials and the probability of success, respectively.

5.51 a. This is not a binomial experiment because there are more than two outcomes for each repetition.

b. This is an example of a binomial experiment because it satisfies all four conditions of a binomial experiment:

i. There are many identical rolls of the die.

ii. Each trial has two outcomes: an even number and an odd number.

iii. The probability of obtaining an even number is 1/2 and that of an odd number is 1/2. These probabilities add up to 1, and they remain constant for all trials.

iv. All rolls of the die are independent.

c. This is an example of a binomial experiment because it satisfies all four conditions of a binomial experiment:

i There are a few identical trials (selection of voters).

ii. Each trial has two outcomes: a voter favors the proposition and a voter does not favor the proposition.

iii. The probability of the two outcomes are .54 and .46, respectively. These probabilities add up to 1. These two probabilities remain the same for all selections.

iv. All voters are independent.

5.52 a. This is an example of a binomial experiment because it satisfies all four conditions of a binomial experiment.

i. There are three identical trials (selections).

ii. Each trial has two outcomes: a red ball is drawn and a blue ball is drawn.

iii. The probability of drawing a red ball is 6/10 and that of a blue ball is 4/10. These probabilities add up to 1. The two probabilities remain constant for all draws because the draws are made with replacement.

iv. All draws are independent.

b. This is not a binomial experiment because the draws are not independent since the selections are made without replacement and, hence, the probabilities of drawing a red and a blue ball change with every selection.

c. This is an example of a binomial experiment because it satisfies all four conditions of a binomial experiment:

i There are a few identical trials (selection of households).

ii. Each trial has two outcomes: a household holds stocks and a household does not hold stocks.

iii. The probabilities of these two outcomes are .28 and .72, respectively. These probabilities add up to 1. The two probabilities remain the same for all selections.

iv. All households are independent.

5.53 a. $n = 8$, $x = 5$, $n - x = 8 - 5 = 3$, $p = .70$, and $q = 1 - p = 1 - .70 = .30$

$P(x = 5) = {}_nC_x p^x q^{n-x} = {}_8C_5 (.70)^5(.30)^3 = (56)(.16807)(.027) = .2541$

b. $n = 4$, $x = 3$, $n - x = 4 - 3 = 1$, $p = .40$, and $q = 1 - p = 1 - .40 = .60$

$P(x = 3) = {}_nC_x p^x q^{n-x} = {}_4C_3 (.40)^3(.60)^1 = (4)(.064)(.60) = .1536$

c. $n = 6$, $x = 2$, $n - x = 6 - 2 = 4$, $p = .30$, and $q = 1 - p = 1 - .30 = .70$

$P(x = 2) = {}_nC_x p^x q^{n-x} = {}_6C_2 (.30)^2(.70)^4 = (15)(.09)(.2401) = .3241$

5.54 a. $n = 5$, $x = 0$, $n - x = 5 - 0 = 5$, $p = .05$, and $q = 1 - p = 1 - .05 = .95$

$P(x = 0) = {}_nC_x p^x q^{n-x} = {}_5C_0 (.05)^0(.95)^5 = (1)(1)(.77378) = .7738$

b. $n = 7$, $x = 4$, $n - x = 7 - 4 = 3$, $p = .90$, and $q = 1 - p = 1 - .90 = .10$

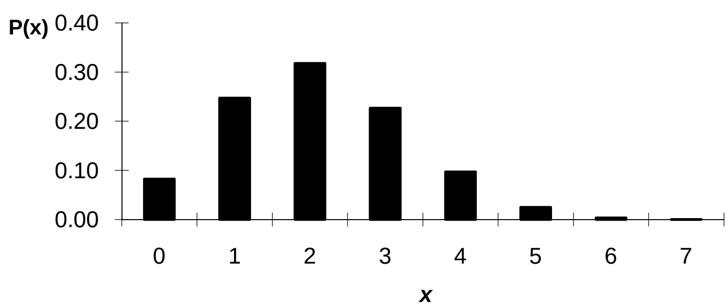
$P(x = 4) = {}_nC_x p^x q^{n-x} = {}_7C_4 (.90)^4(.10)^3 = (35)(.6561)(.001) = .0230$

c. $n = 10$, $x = 7$, $n - x = 10 - 7 = 3$, $p = .60$, and $q = 1 - p = 1 - .60 = .40$

$P(x = 7) = {}_nC_x p^x q^{n-x} = {}_{10}C_7 (.60)^7(.40)^3 = (120)(.0279936)(.064) = .2150$

5.55 a.

x	$P(x)$
0	.0824
1	.2471
2	.3177
3	.2269
4	.0972
5	.0250
6	.0036
7	.0002

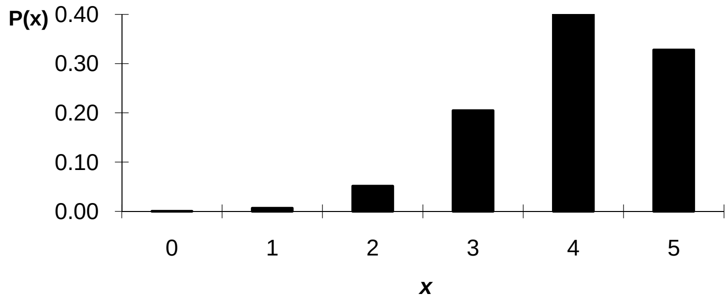


b. $\mu = np = (7)(.30) = 2.100$

$\sigma = \sqrt{npq} = \sqrt{(7)(.30)(.7)} = 1.212$

5.56 a.

x	$P(x)$
0	.0003
1	.0064
2	.0512
3	.2048
4	.4096
5	.3277



b. $\mu = np = (5)(.80) = 4.000$

$\sigma = \sqrt{npq} = \sqrt{(5)(.80)(.20)} = .894$

5.57 Answers will vary depending on the values of n and p selected.

Let $n = 5$. The probability distributions for $p = .30$ (skewed right), $p = .50$ (symmetric), and $p = .80$ (skewed left) are displayed in the tables below followed by the graphs.

***p* = .30**

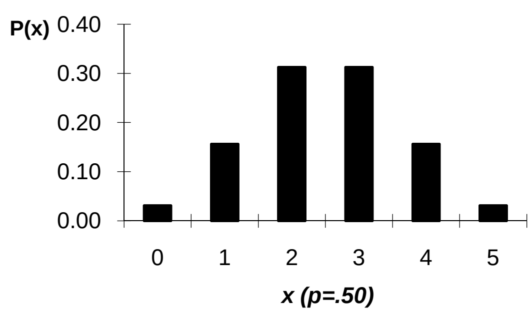
x	*P(x)*
0	.1681
1	.3602
2	.3087
3	.1323
4	.0283
5	.0024

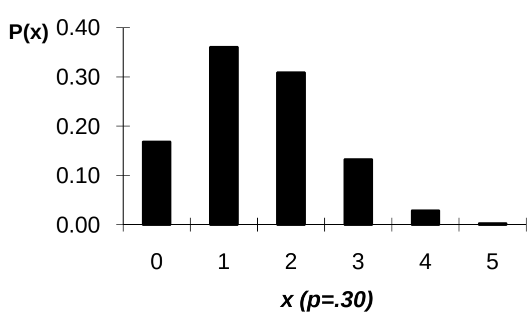
***p* = 0.50**

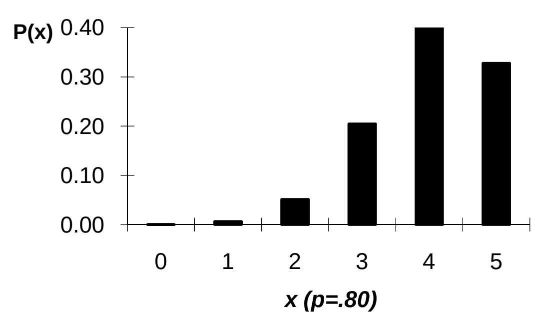
x	*P(x)*
0	.0312
1	.1562
2	.3125
3	.3125
4	.1562
5	.0312

***p* = 0.80**

x	*P(x)*
0	.0003
1	.0064
2	.0512
3	.2048
4	.4096
5	.3277







5.58 a. The random variable x can assume any of the values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, or 15.

b. $n = 15$ and $p = .52$

$P(x = 9) = {}_nC_x p^x q^{n-x} = {}_{15}C_9 (.52)^9(.48)^6 = (5005)(.0027799059)(.0122305905) = .1702$

5.59 a. The random variable x can assume any of the values 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, or 11.

b. $n = 11$ and $p = .46$

$P(x = 3) = {}_nC_x p^x q^{n-x} = {}_{11}C_3 (.46)^3(.54)^8 = (165)(.097336)(.0072301961) = .1161$

5.60 $n = 20$ and $p = .50$

Let x denote the number of adults in a random sample of 20 who said that despite tough economic times they will be willing to pay more for products that have social and environmental benefits.

a. $P(\text{at most } 7) = P(x \le 7) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7)$

$= .0000 + .0000 + .0002 + .0011 + .0046 + .0148 + .0370 + .0739 = .1316$

b. $P(\text{at least } 13) = P(x \ge 13) = P(x = 13) + P(x = 14) + P(x = 15) + P(x = 16) + P(x = 17) + P(x = 18) + P(x = 19) + P(x = 20) = .0739 + .0370 + .0148 + .0046 + .0011 + .0002 + .0000 + .0000 = .1316$

c. $P(12 \text{ to } 15) = P(12 \le x \le 15) = P(x = 12) + P(x = 13) + P(x = 14) + P(x = 15)$

$= .1201 + .0739 + .0370 + .0148 = .2458$

5.61 $n = 14$ and $p = .30$

Let x denote the number of U.S. taxpayers who cheat on their returns.

a. $P(\text{at least } 8) = P(x \ge 8) = P(x = 8) + P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12) + P(x = 13) + P(x = 14) = .0232 + .0066 + .0014 + .0002 + .0000 + .0000 + .0000 = .0314$

b. $P(\text{at most } 3) = P(x \le 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$

$= .0068 + .0407 + .1134 + .1943 = .3552$

c. $P(3 \text{ to } 7) = P(3 \le x \le 7) = P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7)$

$= .1943 + .2290 + .1963 + .1262 + .0618 = .8076$

5.62 $n = 5$ and $p = .20$

Let x denote the number of patients in a random sample of 5 who require sedation.

a. $P(\text{exactly } 2) = P(x = 2) = {}_nC_x p^x q^{n-x} = {}_5C_2 (.20)^2(.80)^3 = (10)(.04)(.512) = .2048$

b. $P(\text{none}) = P(x = 0) = {}_nC_x p^x q^{n-x} = {}_5C_0 (.20)^0(.80)^5 = (1)(1)(.32768) = .3277$

c. $P(\text{exactly } 4) = P(x = 4) = {}_nC_x p^x q^{n-x} = {}_5C_4 (.20)^4(.80)^1 = (5)(.0016)(.8) = .0064$

5.63 $n = 22$ and $p = .65$

Let x denote the number of Americans in a random sample of 22 who take expired medicines.

a. $P(\text{exactly } 17) = P(x = 17) = {}_nC_x p^x q^{n-x} = {}_{22}C_{17} (.65)^{17}(.35)^5 = (26334)(.000659974)(.005252188) = .0913$

b. $P(\text{none}) = P(x = 0) = {}_nC_x p^x q^{n-x} = {}_{22}C_0\,(.65)^0(.35)^{22} = (1)(1)(.000000000093217) = .0000$

c. $P(\text{exactly } 9) = P(x = 9) = {}_nC_x p^x q^{n-x} = {}_{22}C_9\,(.65)^9(.35)^{13} = (497420)(.020711913)(.0000011827)$
$= .0122$

5.64 $n = 10$ and $p = .25$

$P(x = 0) = {}_nC_x p^x q^{n-x} = {}_{10}C_0\,(.25)^0(.75)^{10} = (1)(1)(.0563135) = .0563$

5.65 $n = 8$ and $p = .85$

a. $P(\text{exactly } 8) = P(x = 8) = {}_nC_x p^x q^{n-x} = {}_8C_8\,(.85)^8(.15)^0 = (1)(.27249053)(1) = .2725$

b. $P(\text{exactly } 5) = P(x = 5) = {}_nC_x p^x q^{n-x} = {}_8C_5\,(.85)^5(.15)^3 = (56)(.4437053)(.003375) = .0839$

5.66 $n = 16$ and $p = .70$

Let x denote the number of adults in a random sample of 16 who said that men and women possess equal traits for being leaders.

a. i. $P(\text{exactly } 13) = P(x = 13) = {}_nC_x p^x q^{n-x} = {}_{16}C_{13}\,(.70)^{13}(.30)^3 = (560)(.009688901)(.027) = .1465$

ii. $P(16) = P(x = 16) = {}_nC_x p^x q^{n-x} = {}_{16}C_{16}\,(.70)^{16}(.30)^0 = (1)(.003323293)(1) = .0033$

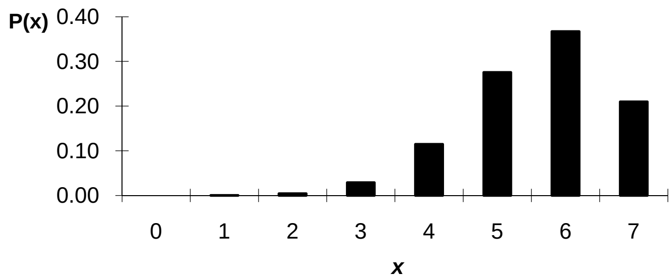
b. i. $P(\text{at least } 11) = P(x \geq 11) = P(x = 11) + P(x = 12) + P(x = 13) + P(x = 14) + P(x = 15)$
$+ P(x = 16) = .2099 + .2040 + .1465 + .0732 + .0228 + .0033 = .6597$

ii. $P(\text{at most } 8) = P(x \leq 8) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)$
$+ P(x = 5) + P(x = 6) + P(x = 7) + P(x = 8)$
$= .0000 + .0000 + .0000 + .0000 + .0002 + .0013 + .0056 + .0185 + .0487 = .0743$

iii. $P(9 \text{ to } 12) = P(9 \leq x \leq 12) = P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12)$
$= .1010 + .1649 + .2099 + .2040 = .6798$

5.67 a. $n = 7$ and $p = .80$

x	$P(x)$
0	.0000
1	.0004
2	.0043
3	.0287
4	.1147
5	.2753
6	.3670
7	.2097



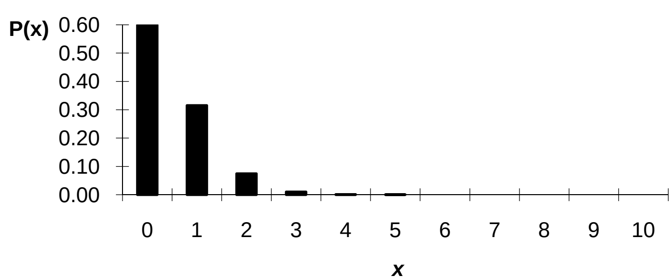
$\mu = np = (7)(.80) = 5.6$ customers

$\sigma = \sqrt{npq} = \sqrt{(7)(.80)(.20)} = 1.058$ customers

b. $P(\text{exactly } 4) = P(x = 4) = .1147$

5.68 a. $n = 10$ and $p = .05$

x	$P(x)$
0	.5987
1	.3151
2	.0746
3	.0105
4	.0010
5	.0001
6	.0000
7	.0000
8	.0000
9	.0000
10	.0000



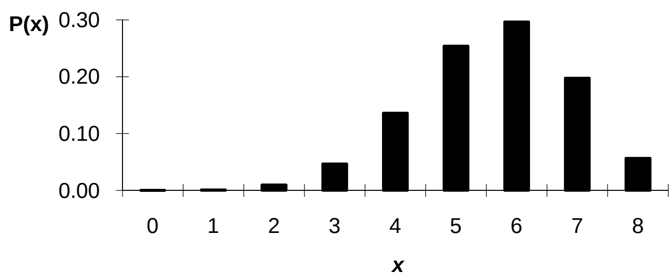
$\mu = np = (10)(.05) = .50$ calculator

$\sigma = \sqrt{npq} = \sqrt{(10)(.05)(.95)} = .689$ calculator

b. $P(\text{exactly } 2) = P(x = 2) = .0746$

5.69 a. $n = 8$ and $p = .70$

x	$P(x)$
0	.0001
1	.0012
2	.0100
3	.0467
4	.1361
5	.2541
6	.2965
7	.1977
8	.0576



$\mu = np = (8)(.70) = 5.600$ customers

$\sigma = \sqrt{npq} = \sqrt{(8)(.70)(.30)} = 1.296$ customers

b. $P(\text{exactly } 3) = P(x = 3) = .0467$

Section 5.7

5.70 The **hypergeometric probability distribution** gives probabilities for the number of successes in a fixed number of trials. It is used for sampling without replacement from a finite population since the trials are not independent. Example 5-23 in the text is an example of the application of a hypergeometric probability distribution.

5.71 $N = 8$, $r = 3$, $N - r = 5$, and $n = 4$

a. $P(x = 2) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_3C_2 \; {}_5C_2}{{}_8C_4} = \dfrac{(3)\,(10)}{70} = .4286$

b. $P(x = 0) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_3C_0 \; {}_5C_4}{{}_8C_4} = \dfrac{(1)(5)}{70} = .0714$

c. $P(x \le 1) = P(x = 0) + P(x = 1) = .0714 + \dfrac{{}_3C_1 \; {}_5C_3}{{}_8C_4} = .0714 + \dfrac{(3)(10)}{70} = .0714 + .4286 = .5000$

5.72 $N = 14$, $r = 6$, $N - r = 8$, and $n = 5$

a. $P(x = 4) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_6C_4 \; {}_8C_1}{{}_{14}C_5} = \dfrac{(15)(8)}{2002} = .0599$

b. $P(x=5)=\frac{{}_rC_x\ {}_{N-r}C_{n-x}}{{}_NC_n}=\frac{{}_6C_5\ {}_8C_0}{{}_{14}C_5}=\frac{(6)(1)}{2002}=.0030$

c. $P(x\le 1)=P(x=0)+P(x=1)=\frac{{}_6C_0\ {}_8C_5}{{}_{14}C_5}+\frac{{}_6C_1\ {}_8C_4}{{}_{14}C_5}=\frac{(1)(56)}{2002}+\frac{(6)(70)}{2002}$

$= .0280 + .2098 = .2378$

5.73 $N = 11$, $r = 4$, $N - r = 7$, and $n = 4$

a. $P(x=2)=\frac{{}_rC_x\ {}_{N-r}C_{n-x}}{{}_NC_n}=\frac{{}_4C_2\ {}_7C_2}{{}_{11}C_4}=\frac{(6)(21)}{330}=.3818$

b. $P(x=4)=\frac{{}_rC_x\ {}_{N-r}C_{n-x}}{{}_NC_n}=\frac{{}_4C_4\ {}_7C_0}{{}_{11}C_4}=\frac{(1)(1)}{330}=.0030$

c. $P(x\le 1)=P(x=0)+P(x=1)=\frac{{}_4C_0\ {}_7C_4}{{}_{11}C_4}+\frac{{}_4C_1\ {}_7C_3}{{}_{11}C_4}=\frac{(1)(35)}{330}+\frac{(4)(35)}{330}=.1061+.4242=.5303$

5.74 $N = 16$, $r = 10$, $N - r = 6$, and $n = 5$

a. $P(x=5)=\frac{{}_rC_x\ {}_{N-r}C_{n-x}}{{}_NC_n}=\frac{{}_{10}C_5\ {}_6C_0}{{}_{16}C_5}=\frac{(252)(1)}{4368}=.0577$

b. $P(x=0)=\frac{{}_rC_x\ {}_{N-r}C_{n-x}}{{}_NC_n}=\frac{{}_{10}C_0\ {}_6C_5}{{}_{16}C_5}=\frac{(1)(6)}{4368}=.0014$

c. $P(x\le 1)=P(x=0)+P(x=1)=.0014+\frac{{}_{10}C_1\ {}_6C_4}{{}_{16}C_5}=.0014+\frac{(10)(15)}{4368}=.0014+.0343=.0357$

5.75 $N = 15$, $r = 9$, $N - r = 6$, and $n = 3$

Let x be the number of corporations that incurred losses in a random sample of 3 corporations, and r be the number of corporations in 15 that incurred losses.

a. P(exactly 2) = $P(x=2)=\frac{{}_rC_x\ {}_{N-r}C_{n-x}}{{}_NC_n}=\frac{{}_9C_2\ {}_6C_1}{{}_{15}C_3}=\frac{(36)(6)}{455}=.4747$

b. P(none) = $P(x=0)=\frac{{}_rC_x\ {}_{N-r}C_{n-x}}{{}_NC_n}=\frac{{}_9C_0\ {}_6C_3}{{}_{15}C_3}=\frac{(1)(20)}{455}=.0440$

c. P(at most 1) = $P(x \le 1) = P(x = 0) + P(x = 1) =$

$=.0440+\frac{{}_9C_1\ {}_6C_2}{{}_{15}C_3}=.0440+\frac{(9)(15)}{455}=.0440+.2967=.3407$

5.76 $N = 20$, $r = 4$, $N - r = 16$, and $n = 6$

Let x be the number of jurors acquainted with one or more of the litigants in a random sample of 6 jurors, and r be the number of jurors in 20 acquainted with one or more of the litigants.

a. P(exactly 1) = $P(x=1)=\frac{{}_rC_x\ {}_{N-r}C_{n-x}}{{}_NC_n}=\frac{{}_4C_1\ {}_{16}C_5}{{}_{20}C_6}=\frac{(4)(4368)}{38{,}760}=.4508$

b. $P(\text{none}) = P(x=0) = \dfrac{{}_{r}C_{x}\ {}_{N-r}C_{n-x}}{{}_{N}C_{n}} = \dfrac{{}_{4}C_{0}\ {}_{16}C_{6}}{{}_{20}C_{6}} = \dfrac{(1)(8008)}{38{,}760} = .2066$

c. $P(\text{at most 2}) = P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$

$= .2066 + .4508 + \dfrac{{}_{4}C_{2}\ {}_{16}C_{4}}{{}_{20}C_{6}} = .2066 + .4508 + \dfrac{(6)(1820)}{38{,}760} = .2066 + .4508 + .2817 = .9391$

5.77 $N = 18$, $r = 11$, and $N - r = 7$, and $n = 4$

Let x be the number of unspoiled eggs in a random sample of 4, and r be the number of unspoiled eggs in 18.

a. $P(\text{exactly 4}) = P(x=4) = \dfrac{{}_{r}C_{x}\ {}_{N-r}C_{n-x}}{{}_{N}C_{n}} = \dfrac{{}_{11}C_{4}\ {}_{7}C_{0}}{{}_{18}C_{4}} = \dfrac{(330)(1)}{3060} = .1078$

b. $P(\text{2 or fewer}) = P(x \le 2) = P(x=0) + P(x=1) + P(x=2)$

$= \dfrac{{}_{11}C_{0}\ {}_{7}C_{4}}{{}_{18}C_{4}} + \dfrac{{}_{11}C_{1}\ {}_{7}C_{3}}{{}_{18}C_{4}} + \dfrac{{}_{11}C_{2}\ {}_{7}C_{2}}{{}_{18}C_{4}} = \dfrac{(1)(35)}{3060} + \dfrac{(11)(35)}{3060} + \dfrac{(55)(21)}{3060} = .0114 + .1258 + .3775 = .5147$

c. $P(\text{more than 1}) = P(x > 1) = P(x=2) + P(x=3) + P(x=4)$

$= .3775 + \dfrac{{}_{11}C_{3}\ {}_{7}C_{1}}{{}_{18}C_{4}} + .1078 = .3775 + \dfrac{(165)(7)}{3060} + .1078 = .3775 + .3775 + .1078 = .8628$

5.78 $N = 20$, $r = 6$, $N - r = 14$, and $n = 5$

Let x be the number of defective keyboards in a sample of 5 from the box selected, and r be the number of defective keyboards in 20.

a. $P(\text{shipment accepted}) = P(x \le 1) = P(x=0) + P(x=1)$

$= \dfrac{{}_{6}C_{0}\ {}_{14}C_{5}}{{}_{20}C_{5}} + \dfrac{{}_{6}C_{1}\ {}_{14}C_{4}}{{}_{20}C_{5}} = \dfrac{(1)(2002)}{15{,}504} + \dfrac{(6)(1001)}{15{,}504} = .1291 + .3874 = .5165$

b. $P(\text{shipment not accepted}) = 1 - P(\text{shipment accepted}) = 1 - .5165 = .4835$

Section 5.8

5.79 The following three conditions must be satisfied to apply the Poisson probability distribution:

1) x is a discrete random variable.
2) The occurrences are random.
3) The occurrences are independent.

5.80 The parameter of the Poisson probability distribution is λ, which represents the mean number of occurrences in an interval.

5.81 a. $P(x \le 1) = P(x=0) + P(x=1) = \dfrac{(5)^0 e^{-5}}{0!} + \dfrac{(5)^1 e^{-5}}{1!} = \dfrac{(1)(.00673795)}{1} + \dfrac{(5)(.00673795)}{1}$

$= .0067 + .0337 = .0404$

Note that the value of e^{-5} is obtained from Table II of Appendix C of the text.

b. $P(x=2)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{(2.5)^2 e^{-2.5}}{2!}=\frac{(6.25)(.08208500)}{2}=.2565$

5.82 a. $P(x<2)=P(x=0)+P(x=1)=\frac{(3)^0 e^{-3}}{0!}+\frac{(3)^1 e^{-3}}{1!}=\frac{(1)(.04978707)}{1}+\frac{(3)(.04978707)}{1}$

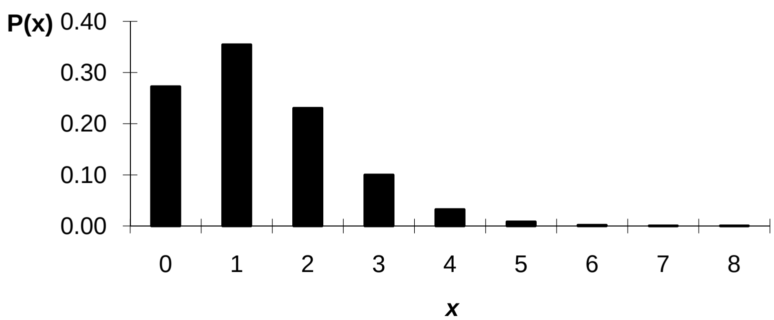
$= .0498 + .1494 = .1992$

Note that the value of e^{-3} is obtained form Table II of Appendix C of the text.

b. $P(x=8)=\frac{\lambda^x e^{-\lambda}}{x!}=\frac{(5.5)^8 e^{-5.5}}{8!}=\frac{(837{,}339.3789)(.00408677)}{40{,}320}=.0849$

5.83 a.

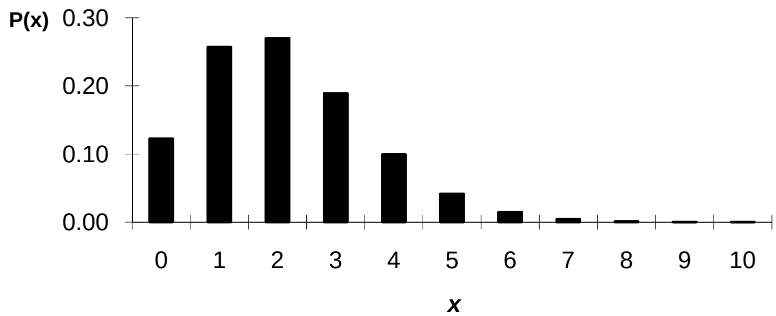
x	$P(x)$
0	.2725
1	.3543
2	.2303
3	.0998
4	.0324
5	.0084
6	.0018
7	.0003
8	.0001



$\mu=\lambda=1.3$, $\sigma^2=\lambda=1.3$, and $\sigma=\sqrt{\lambda}=\sqrt{1.3}=1.140$

b.

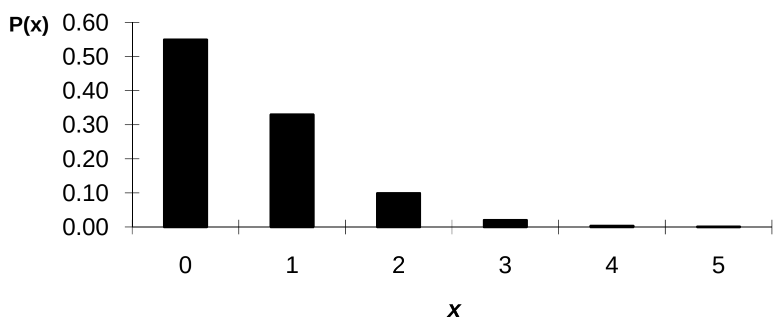
x	$P(x)$
0	.1225
1	.2572
2	.2700
3	.1890
4	.0992
5	.0417
6	.0146
7	.0044
8	.0011
9	.0003
10	.0001



$\mu = \lambda = 2.1, \sigma^2 = \lambda = 2.1$, and $\sigma = \sqrt{\lambda} = \sqrt{2.1} = 1.449$

5.84 a.

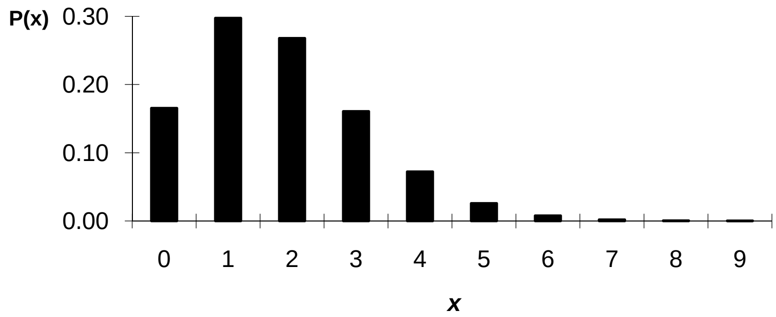
x	$P(x)$
0	.5488
1	.3293
2	.0988
3	.0198
4	.0030
5	.0004



$\mu = \lambda = .6, \sigma^2 = \lambda = .6$, and $\sigma = \sqrt{\lambda} = \sqrt{.6} = .775$

b.

x	$P(x)$
0	.1653
1	.2975
2	.2678
3	.1607
4	.0723
5	.0260
6	.0078
7	.0020
8	.0005
9	.0001



$\mu = \lambda = 1.8$, $\sigma^2 = \lambda = 1.8$, and $\sigma = \sqrt{\lambda} = \sqrt{1.8} = 1.342$

5.85 λ = 1.7 pieces of junk mail per day

$$P(\text{exactly three}) = P(x=3) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(1.7)^3 e^{-1.7}}{3!} = \frac{(4.913)(.18268352)}{6} = .1496$$

5.86 λ = 9.7 complaints per day

$$P(\text{exactly six}) = P(x=6) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(9.7)^6 e^{-9.7}}{6!} = \frac{(832{,}972.0049)(.00006128)}{720} = .0709$$

5.87 λ = 5.4 shoplifting incidents per day

$$P(\text{exactly three}) = P(x=3) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(5.4)^3 e^{-5.4}}{3!} = \frac{(157.464)(.00451658)}{6} = .1185$$

5.88 λ = 12.5 rooms per day

$$P(\text{exactly three}) = P(x=3) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(12.5)^3 e^{-12.5}}{3!} = \frac{(1953.125)(.00000373)}{6} = .0012$$

5.89 λ = 3.7 reports of lost students' ID cards per week

a. $P(\text{at most } 1) = P(x \le 1) = P(x=0) + P(x=1)$

$$= \frac{(3.7)^0 e^{-3.7}}{0!} + \frac{(3.7)^1 e^{-3.7}}{1!} = \frac{(1)(.02472353)}{1} + \frac{(3.7)(.02472353)}{1} = .0247 + .0915 = .1162$$

b. i. $P(1 \text{ to } 4) = P(1 \le x \le 4) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= .0915 + .1692 + .2087 + .1931 = .6625$

ii. $P(\text{at least } 6) = P(x \ge 6)$

$= P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10) + P(x=11) + P(x=12) + P(x=13)$

$= .0881 + .0466 + .0215 + .0089 + .0033 + .0011 + .0003 + .0001 = .1699$

iii. $P(\text{at most } 3) = P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= .0247 + .0915 + .1692 + .2087 = .4941$

5.90 Let x be the number of businesses that file for bankruptcy on a given day in this city. $\lambda = 1.6$ filings per day

a. $P(\text{exactly } 3) = P(x=3) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(1.6)^3 e^{-1.6}}{3!} = \dfrac{(4.096)(.20189652)}{6} = .1378$

b. i. $P(2 \text{ to } 3) = P(2 \le x \le 3) = P(x=2) + P(x=3) = .2584 + .1378 = .3962$

ii. $P(\text{more than } 3) = P(x > 3) = P(x=4) + P(x=5) + P(x=6) + P(x=7) + P(x=8) + P(x=9)$
$= .0551 + .0176 + .0047 + .0011 + .0002 + .0000 = .0787$

iii. $P(\text{less than } 3) = P(x < 3) = P(x=0) + P(x=1) + P(x=2) = .2019 + .3230 + .2584 = .7833$

5.91 Let x be the number of defects in a given 500-yard piece of fabric. $\lambda = .5$ defect per 500 yards

a. $P(\text{exactly } 1) = P(x=1) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(.5)^1 e^{-.5}}{1!} = \dfrac{(.5)(.60653066)}{1} = .3033$

b. i. $P(2 \text{ to } 4) = P(2 \le x \le 4) = P(x=2) + P(x=3) + P(x=4) = .0758 + .0126 + .0016 = .0900$

ii. $P(\text{more than } 3) = P(x > 3) = P(x=4) + P(x=5) + P(x=6) + P(x=7)$
$= .0016 + .0002 + .0000 + .0000 = .0018$

iii. $P(\text{less than } 2) = P(x < 2) = P(x=0) + P(x=1) = .6065 + .3033 = .9098$

5.92 Let x be the number of students who login to a randomly selected computer in a college computer lab per day. $\lambda = 19$ students per day

a. $P(\text{exactly } 12) = P(x=12)$

$$= \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(19)^{12} e^{-19}}{12!} = \frac{(2213310000000000)(.0000000056028)}{479001600} = .0259$$

b. i. $P(13 \text{ to } 16) = P(13 \le x \le 16) = P(x=13) + P(x=14) + P(x=15) + P(x=16)$
$= .0378 + .0514 + .0650 + .0772 = .2314$

ii. $P(\text{fewer than } 8) = P(x < 8) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5)$
$+ P(x=6) + P(x=7)$
$= .0000 + .0000 + .0000 + .0000 + .0000 + .0001 + .0004 + .0010 = .0015$

5.93 Let x be the number of customers that come to this savings and loan during a given hour. Since the average number of customers per half hour is 4.8, $\lambda = (2)(4.8) = 9.6$ customers per hour.

a. $P(\text{exactly } 2) = P(x=2) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(9.6)^2 e^{-9.6}}{2!} = \dfrac{(92.16)(.00006773)}{2} = .0031$

b. i. $P(2 \text{ or fewer}) = P(x \le 2) = P(x=0) + P(x=1) + P(x=2) = .0001 + .0007 + .0031 = .0039$

ii. $P(10 \text{ or more}) = P(x \ge 10) = P(x=10) + P(x=11) + P(x=12) + \ldots + P(x=24)$
$= .1241 + .1083 + .0866 + .0640 + .0439 + .0281 + .0168 + .0095 + .0051 + .0026 + .0012$
$+ .0006 + .0002 + .0001 + .0000 = .4911$

5.94 $\lambda = 3.2$ unsolicited applications per week

a. $P(\text{none}) = P(x=0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(3.2)^0 e^{-3.2}}{0!} = \dfrac{(1)(.04076220)}{1} = .0408$

b.

x	$P(x)$
0	.0408
1	.1304
2	.2087
3	.2226
4	.1781
5	.1140
6	.0608
7	.0278
8	.0111
9	.0040
10	.0013
11	.0004
12	.0001

c. $\mu = \lambda = 3.2$, $\sigma^2 = \lambda = 3.2$, and $\sigma = \sqrt{\lambda} = \sqrt{3.2} = 1.789$

5.95 Let x be the number of policies sold by this salesperson on a given day. $\lambda = 1.4$ policies per day

a. $P(\text{none}) = P(x=0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(1.4)^0 e^{-1.4}}{0!} = \dfrac{(1)(.24659696)}{1} = .2466$

b.

x	$P(x)$
0	.2466
1	.3452
2	.2417
3	.1128
4	.0395
5	.0111
6	.0026
7	.0005
8	.0001

c. $\mu = \lambda = 1.4$, $\sigma^2 = \lambda = 1.4$, and $\sigma = \sqrt{\lambda} = \sqrt{1.4} = 1.183$

5.96 Let x denote the number of accidents on a given day. $\lambda = .8$ accident per day

a. $P(\text{none}) = P(x=0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(.8)^0 e^{-.8}}{0!} = \dfrac{(1)(.44932896)}{1} = .4493$

b.

x	$P(x)$
0	.4493
1	.3595
2	.1438
3	.0383
4	.0077
5	.0012
6	.0002

c. $\mu = \lambda = .8$, $\sigma^2 = \lambda = .8$, and $\sigma = \sqrt{\lambda} = \sqrt{.8} = .894$

5.97 Let x denote the number of households in a random sample of 50 who own answering machines.

$\lambda = 20$ households in 50

a. $P(\text{exactly } 25) = P(x = 25) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(20)^{25} e^{-20}}{25!} = .0446$

b. i. $P(\text{at most } 12) = P(x \le 12) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$
$+ P(x = 6) + P(x = 7) + P(x = 8) + P(x = 9) + P(x = 10) + P(x = 11)$
$+ P(x = 12) = .0000 + .0000 + .0000 + .0000 + .0000 + .0001 + .0002 + .0005 + .0013 + .0029$
$+ .0058 + .0106 + .0176 = .0390$

ii. $P(13 \text{ to } 17) = P(13 \le x \le 17) = P(x = 13) + P(x = 14) + P(x = 15) + P(x = 16) + P(x = 17)$
$= .0271 + .0387 + .0516 + .0646 + .0760 = .2580$

iii. $P(\text{at least } 30) = P(x \ge 30) = P(x = 30) + P(x = 31) + P(x = 32) + P(x = 33) + P(x = 34)$
$+ P(x = 35) + P(x = 36) + P(x = 37) + P(x = 38) + P(x = 39)$
$= .0083 + .0054 + .0034 + .0020 + .0012 + .0007 + .0004 + .0002 + .0001 + .0001 = .0218$

5.98 Let x denote the number of cars passing through a school zone exceeding the speed limit. The average number of cars speeding by at least ten miles per hour is 20 percent. Thus, $\lambda = (.20)(100) = 20$ cars per 100.

a. $P(\text{exactly } 25) = P(x = 25) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(20)^{25} e^{-20}}{25!} = .0446$

b. i. $P(\text{at most } 8) = P(x \le 8) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$
$+ P(x = 6) + P(x = 7) + P(x = 8)$
$= .0000 + .0000 + .0000 + .0000 + .0000 + .0001 + .0002 + .0005 + .0013 = .0021$

ii. $P(15 \text{ to } 20) = P(15 \le x \le 20)$
$= P(x = 15) + P(x = 16) + P(x = 17) + P(x = 18) + P(x = 19) + P(x = 20)$
$= .0516 + .0646 + .0760 + .0844 + .0888 + .0888 = .4542$

iii. $P(\text{at least } 30) = P(x \ge 30) = P(x = 30) + P(x = 31) + P(x = 32) + P(x = 33) + P(x = 34)$
$+ P(x = 35) + P(x = 36) + P(x = 37) + P(x = 38) + P(x = 39) = .0083 + .0054 + .0034 + .0020 +$
$.0012 + .0007 + .0004 + .0002 + .0001 + .0001 = .0218$

Supplementary Exercises

5.99

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
2	.05	.10	4	.20
3	.22	.66	9	1.98
4	.40	1.60	16	6.40
5	.23	1.15	25	5.75
6	.10	.60	36	3.60
		$\sum xP(x) = 4.11$		$\sum x^2P(x) = 17.93$

$\mu = \sum xP(x) = 4.11$ cars

$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{17.93 - (4.11)^2} = 1.019$ cars

This mechanic repairs, on average, 4.11 cars per day.

5.100

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.13	.00	0	.00
1	.28	.28	1	.28
2	.30	.60	4	1.20
3	.17	.51	9	1.53
4	.08	.32	16	1.28
5	.04	.20	25	1.00
		$\sum xP(x) = 1.91$		$\sum x^2P(x) = 5.29$

$\mu = \sum xP(x) = 1.91$ root canals

$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{5.29 - (1.91)^2} = 1.281$ root canals

Dr. Sharp performs an average of 1.91 root canals on Monday.

5.101 a.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
-1.2	.17	-.204	1.44	.2448
-.7	.21	-.147	.49	.1029
.9	.37	.333	.81	.2997
2.3	.25	.575	5.29	1.3225
		$\sum xP(x) = .557$		$\sum x^2P(x) = 1.9699$

b. $\mu = \sum xP(x) = \$.557 \text{ million} = \$557{,}000$

$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{1.9699 - (557)^2} = \$1.288274 \text{ million} = \$1{,}288{,}274$

The company has an expected profit of $557,000 for next year.

5.102 a. Note that if the policyholder dies next year, the company's net loss of $100,000 is offset by the $350 premium. Thus, in this case, $x = 350 - 100{,}000 = -99{,}650$.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
−99,650	.002	−199.30	9,930,122,500	19,860,245
350	.998	349.30	122,500	122,255
		$\sum xP(x)$ = 150.00		$\sum x^2P(x)$ = 19,982,500

b. $\mu = \sum xP(x) = \$150.00$

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{19{,}982{,}500 - (150.00)^2} = \$4467.66$

The company's expected gain for the next year on this policy is $150.00.

5.103 Let x denote the number of machines that are broken down at a given time. Assuming machines are independent, x is a binomial random variable with $n = 8$ and $p = .04$.

a. $P(\text{exactly } 8) = P(x = 8) = {}_nC_x p^x q^{n-x} = {}_8C_8\,(.04)^8(.96)^0 = (1)(.000000000007)(1) \approx .0000$

b. $P(\text{exactly } 2) = P(x = 2) = {}_nC_x p^x q^{n-x} = {}_8C_2\,(.04)^2(.96)^6 = (28)(.0016)(.78275779) = .0351$

c. $P(\text{none}) = P(x = 0) = {}_nC_x p^x q^{n-x} = {}_8C_0\,(.04)^0(.96)^8 = (1)(1)(.72138958) = .7214$

5.104 Let x denote the number of the 12 new credit card holders who will eventually default. Then x is a binomial random variable with $n = 12$ and $p = .08$.

a. $P(\text{exactly } 3) = P(x = 3) = {}_nC_x p^x q^{n-x} = {}_{12}C_3\,(.08)^3(.92)^9 = (220)(.000512)(.47216136) = .0532$

b. $P(\text{exactly } 1) = P(x = 1) = {}_nC_x p^x q^{n-x} = {}_{12}C_1\,(.08)^1(.92)^{11} = (12)(.08)(.39963738) = .3837$

c. $P(\text{none}) = P(x = 0) = {}_nC_x p^x q^{n-x} = {}_{12}C_0\,(.08)^0(.92)^{12} = (1)(1)(.36766639) = .3677$

5.105 Let x denote the number of defective motors in a random sample of 20. Then x is a binomial random variable with $n = 20$ and $p = .05$.

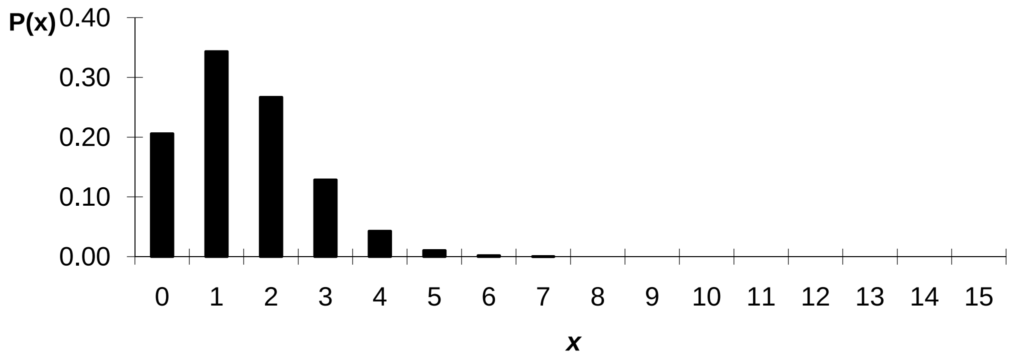
a. $P(\text{shipment accepted}) = P(x \le 2) = P(x = 0) + P(x = 1) + P(x = 2) = .3585 + .3774 + .1887 = .9246$

b. $P(\text{shipment rejected}) = 1 - P(\text{shipment accepted}) = 1 - .9246 = .0754$

5.106 a. $n = 15$ and $p = .10$

x	$P(x)$
0	.2059
1	.3432
2	.2669
3	.1285
4	.0428
5	.0105
6	.0019
7	.0003

Note that the probabilities of $x = 8$ to $x = 15$ are all equal to .0000 from Table I of Appendix C.



$\mu = np = (15)(.10) = 1.5$ returns

$\sigma = \sqrt{npq} = \sqrt{(15)(.10)(.90)} = 1.162$ returns

b. $P(\text{exactly } 5) = P(x = 5) = .0105$

5.107 Let x denote the number of households who own homes in the random sample of 4 households. Then x is a hypergeometric random variable with $N = 15$, $r = 9$, $N - r = 6$ and $n = 4$.

a. $P(\text{exactly } 3) = P(x=3) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_9C_3 \; {}_6C_1}{{}_{15}C_4} = \dfrac{(84)(6)}{1365} = .3692$

b. $P(\text{at most } 1) = P(x \le 1) = P(x = 0) + P(x = 1)$

$= \dfrac{{}_9C_0 \; {}_6C_4}{{}_{15}C_4} + \dfrac{{}_9C_1 \; {}_6C_3}{{}_{15}C_4} = \dfrac{(1)(15)}{1365} + \dfrac{(9)(20)}{1365} = .0110 + .1319 = .1429$

c. $P(\text{exactly } 4) = P(x=4) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_9C_4 \; {}_6C_0}{{}_{15}C_4} = \dfrac{(126)(1)}{1365} = .0923$

5.108 Let x denote the number of corporations in the random sample of 5 that provide retirement benefits. Then x is a hypergeometric random variable with $N = 20$, $r = 14$, $N - r = 6$, and $n = 5$.

a. $P(\text{exactly } 2) = P(x=2) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_{14}C_2 \; {}_6C_3}{{}_{20}C_5} = \dfrac{(91)(20)}{15{,}504} = .1174$

b. $P(\text{none}) = P(x=0) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_{14}C_0 \; {}_6C_5}{{}_{20}C_5} = \dfrac{(1)(6)}{15{,}504} = .0004$

c. $P(\text{at most one}) = P(x \le 1) = P(x = 0) + P(x = 1)$

$= .0004 + \dfrac{{}_{14}C_1 \; {}_6C_4}{{}_{20}C_5} = .0004 + \dfrac{(14)(15)}{15{,}504} = .0004 + .0135 = .0139$

5.109 Let x denote the number of defective parts in a random sample of 4. Then x is a hypergeometric random variable with $N = 16$, $r = 3$, $N - r = 13$, and $n = 4$.

a. P(shipment accepted) = $P(x \le 1) = P(x = 0) + P(x = 1)$

$$= \frac{{}_3C_0 \; {}_{13}C_4}{{}_{16}C_4} + \frac{{}_3C_1 \; {}_{13}C_3}{{}_{16}C_4} = \frac{(1)(715)}{1820} + \frac{(3)(286)}{1820} = .3929 + .4714 = .8643$$

b. P(shipment not accepted) = 1 – P(shipment accepted) = 1 – .8643 = .1357

5.110 Let x denote the number of tax returns in a random sample of 3 that contain errors. Then x is a hypergeometric random variable with $N = 12$, $r = 2$, $N - r = 10$, and $n = 3$.

a. $P(\text{exactly } 1) = P(x = 1) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_2C_1 \; {}_{10}C_2}{{}_{12}C_3} = \dfrac{(2)(45)}{220} = .4091$

b. $P(\text{none}) = P(x = 0) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_2C_0 \; {}_{10}C_3}{{}_{12}C_3} = \dfrac{(1)(120)}{220} = .5455$

c. $P(\text{exactly } 2) = P(x = 2) = \dfrac{{}_rC_x \; {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_2C_2 \; {}_{10}C_1}{{}_{12}C_3} = \dfrac{(1)(10)}{220} = .0455$

5.111 $\lambda = 7$ cases per day

a. $P(\text{exactly } 4) = P(x = 4) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(7)^4 e^{-7}}{4!} = \dfrac{(2401)(.00091188)}{24} = .0912$

b. i. P(at least 7) = $P(x \ge 7) = P(x = 7) + P(x = 8) + P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12)$
$+ P(x = 13) + P(x = 14) + P(x = 15) + P(x = 16) + P(x = 17) + P(x = 18)$
= .1490 + .1304 + .1014 + .0710 + .0452 + .0263 + .0142 + .0071 + .0033 + .0014 + .0006
+ .0002 + .0001 = .5502

ii. P(at most 3) = $P(x \le 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$
= .0009 + .0064 + .0223 + .0521 = .0817

iii. P(2 to 5) = $P(2 \le x \le 5) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$
= .0223 + .0521 + .0912 + .1277 = .2933

5.112 $\lambda = 6.3$ robberies per day

a. $P(\text{exactly } 3) = P(x = 3) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(6.3)^3 e^{-6.3}}{3!} = \dfrac{(250.047)(.00183630)}{6} = .0765$

b. i. P(at least 12) = $P(x \ge 12) = P(x = 12) + P(x = 13) + P(x = 14) + P(x = 15) + P(x = 16)$
$+ P(x = 17) + P(x = 18) + P(x = 19)$
= .0150 + .0073 + .0033 + .0014 + .0005 + .0002 + .0001 + .0000 = .0278

ii. P(at most 3) = $P(x \le 3) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$
= .0018 + .0116 + .0364 + .0765 = .1263

iii. P(2 to 6) = $P(2 \le x \le 6) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6)$
= .0364 + .0765 + .1205 + .1519 + .1595 = .5448

5.113 $\lambda = 1.4$ airplanes per hour

a. $P(\text{none}) = P(x = 0) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(1.4)^0 e^{-1.4}}{0!} = \dfrac{(1)(.24659696)}{1} = .2466$

b.

x	$P(x)$
0	.2466
1	.3452
2	.2417
3	.1128
4	.0395
5	.0111
6	.0026
7	.0005
8	.0001

5.114 $\lambda = 1.2$ technical fouls per game

a. $P(\text{exactly } 3) = P(x = 3) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(1.2)^3 e^{-1.2}}{3!} = \dfrac{(1.728)(.30119421)}{6} = .0867$

b.

x	$P(x)$
0	.3012
1	.3614
2	.2169
3	.0867
4	.0260
5	.0062
6	.0012
7	.0002

5.115 Let x be a random variable that denotes the gain you have from this game. There are 36 different outcomes for two dice: (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2),…, (6, 6).

$P(sum = 2) = P(sum = 12) = \dfrac{1}{36}$ $\qquad$ $P(sum = 3) = P(sum = 11) = \dfrac{2}{36}$

$P(sum = 4) = P(sum = 10) = \dfrac{3}{36}$ $\qquad$ $P(sum = 9) = \dfrac{4}{36}$

$P(x = 20) = P(\text{you win}) = P(sum = 2) + P(sum = 3) + P(sum = 4) + P(sum = 9) + P(sum = 10)$

$+ P(sum = 11) + P(sum = 12) = \dfrac{1}{36} + \dfrac{2}{36} + \dfrac{3}{36} + \dfrac{4}{36} + \dfrac{3}{36} + \dfrac{2}{36} + \dfrac{1}{36} = \dfrac{16}{36}$

$P(x = -20) = P(\text{you lose}) = 1 - P(\text{you win}) = 1 - \dfrac{16}{36} = \dfrac{20}{36}$

x	$P(x)$	$xP(x)$
20	$\frac{16}{36} = .4444$	8.89
–20	$\frac{20}{36} = .5556$	–11.11
		$\sum xP(x) = -2.22$

The value of $\sum xP(x) = -2.22$ indicates that your expected "gain" is –$2.22, so you should not accept this offer. This game is not fair to you since you are expected to lose $2.22.

5.116 Let x be a random variable that denotes the net profit of the venture, and p denotes the probability for a success.

x	$P(x)$	$xP(x)$
10,000,000	p	$10{,}000{,}000p$
–4,000,000	$1-p$	$-4{,}000{,}000(1-p)$
		$\sum xP(x) = 10{,}000{,}000p - 4{,}000{,}000(1-p)$

$\mu = \sum xP(x) = 10{,}000{,}000p - 4{,}000{,}000(1-p) = 14{,}000{,}000p - 4{,}000{,}000$

a. If $p = .40$, $\mu = 14{,}000{,}000p - 4{,}000{,}000 = 14{,}000{,}000(.40) - 4{,}000{,}000 = \$1{,}600{,}000$.
Yes, the owner will be willing to take the risk as the expected net profit is above $500,000.

b. As long as $\mu \geq 500{,}000$ the owner will be willing to take the risk. This means as long as $\mu = 14{,}000{,}000p - 4{,}000{,}000 \geq 500{,}000$, the owner will do it. This inequality holds for $p \geq .3214$.

5.117 a. Team A needs to win four games in a row, each with probability .5, so
P(team A wins the series in four games) = $.5^4 = .0625$

b. In order to win in five games, Team A needs to win 3 of the first four games as well as the fifth game, so P(team A wins the series in five games) = ${}_4C_3(.5)^3(.5)(.5) = .125$.

c. If seven games are required for a team to win the series, then each team needs to win three of the first six games, so P(seven games are required to win the series) = ${}_6C_3(.5)^3(.5)^3 = .3125$.

5.118 Let x denote the number of bearings in a random sample of 15 that do not meet the required specifications. Then x is a binomial random variable with $n = 15$ and $p = .10$.

a. P(production suspended) = $P(x > 2) = 1 - P(x \leq 2) = 1 - [P(x = 0) + P(x = 1) + P(x = 2)]$
$= 1 - (.2059 + .3432 + .2669) = .1840$

b. The 15 bearings are sampled without replacement, which normally requires the use of the hypergeometric distribution. We are assuming that the population from which the sample is drawn is so large that each time a bearing is selected, the probability of being defective is .10. Thus, the sampling of 15 bearings constitutes 15 independent trials, so that the distribution of x is approximately binomial.

5.119 a. Let x denote the number of drug deals on this street on a given night. Note that x is discrete. This text has covered two discrete distributions, the binomial and the Poisson. The binomial distribution does not apply here, since there is no fixed number of "trials". However, the Poisson distribution might be appropriate since we have an estimated average number of occurrences over a particular interval.

b. To use the Poisson distribution we would have to assume that the drug deals occur randomly and independently.

c. The mean number of drug deals per night is three; if the residents tape for two nights, then $\lambda = (2)(3) = 6$.

$P(\text{film at least 5 drug deals}) = P(x \geq 5) = 1 - P(x < 5)$

$= 1 - [P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4)]$

$= 1 - (.0025 + .0149 + .0446 + .0892 + .1339) = .7149$

d. Part c. shows that two nights of taping are insufficient, since $P(x \geq 5) = .7149 < .90$. Try taping for three nights; then $\lambda = (3)(3) = 9$.

$P(x \geq 5) = 1 - (.0001 + .0011 + .0050 + .0150 + .0337) = .9451$.

This exceeds the required probability of .90, so the camera should be rented for three nights.

5.120 a. It's easier to do part b first.

b. Let x be a random variable that denotes the score after guessing on one multiple choice question.

x	$P(x)$	$xP(x)$
1	.25	.250
$-\frac{1}{2}$	.75	$-.375$
		$\sum xP(x) = -.125$

So a student decreases his expected score by guessing on a question, if he has no idea what the correct answer is.

Back to part a.: Guessing on 12 questions lowers the expected score by $(12)(-.125) = -1.5$, so the expected score for 38 correct answers and 12 random guesses is $(38)(1) + (12)(-.125) = 36.5$.
If the student can eliminate one of the wrong answers, then we get the following:

x	$P(x)$	$xP(x)$
1	$\frac{1}{3}$	$\frac{1}{3}$
$-\frac{1}{2}$	$\frac{2}{3}$	$-\frac{1}{3}$
		$\sum xP(x) = 0$

So in this case guessing does not affect the expected score.

5.121 Let x be the number of sales per day, and let λ be the mean number of cheesecakes sold per day. Here $\lambda = 5$. We want to find k such that $P(x > k) < .1$. Using the Poisson probability distribution we find that $P(x > 7) = 1 - P(x \leq 7) = 1 - .867 = .133$ and $P(x > 8) = 1 - P(x \leq 8) = 1 - .932 = .068$. So, if the baker wants the probability of losing a sale to be less than .1, he needs to make 8 cheesecakes.

5.122 a. For the \$1 outcome:

P(gambler wins) = 22/50 = .44; net gain = \$1

P(gambler loses) = 1 – (22/50) = .56; net gain = –\$1

Therefore, the expected net payoff for a \$1 bet on the \$1 outcome is (1)(.44) – (1)(.56) = –\$.12 = –12¢. Thus, the gambler has an average net loss of 12¢ per \$1 bet on the \$1 outcome.

b. When the gambler bets \$1 and loses, his loss is \$1 regardless of the outcome on which he/she bet. The expected values for a \$1 bet on each of the other six outcomes are shown in the following table.

Outcome	P(win)	P(lose)	Expected net payoff
\$2	14/50 = .28	.72	(2)(.28) – (1)(.72) = –.16
5	7/50 = .14	.86	(5)(.14) – (1)(.86) = –.16
10	3/50 = .06	.94	(10)(.06) – (1)(.94) = –.34
20	2/50 = .04	.96	(20)(.04) – (1)(.96) = –.16
flag	1/50 = .02	.98	(40)(.02) – (1)(.98) = –.18
joker	1/50 = .02	.98	(40)(.02) – (1)(.98) = –.18

c. In terms of expected net payoff, the \$1 outcome is best (–12¢), while the \$10 outcome is worst (–34¢).

5.123 a. There are ${}_7C_4 = 35$ ways to choose four questions from the set of seven.

b. The teacher must choose both questions that the student did not study, and any two of the remaining five questions. Thus, there are ${}_2C_2\,{}_5C_2$ = (1) (10) = 10 ways to choose four questions that include the two that the student did not study.

c. From the answers to parts a and b,

P(the four questions on the test include both questions the student did not study) = 10/35 = .2857.

5.124 For each game, let x = amount you win

Game I:

Outcome	x	$P(x)$	$xP(x)$	$x^2P(x)$
Head	3	.50	1.50	4.50
Tail	-1	.50	-.50	.50
			$\sum xP(x) = 1.00$	$\sum x^2P(x) = 5.00$

$$\mu = \sum xP(x) = \$1.00$$

$$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{5 - (1)^2} = \$2.00$$

Game II:

Outcome	x	$P(x)$	$xP(x)$	$x^2P(x)$
First ticket	300	1/500	.60	180
Second ticket	150	1/500	.30	45
Neither	0	498/500	.00	0
			$\sum xP(x) = .90$	$\sum x^2P(x) = 225$

$$\mu = \sum xP(x) = \$.90$$

$$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{225 - (.90)^2} = \$14.97$$

Game III:

Outcome	x	$P(x)$	$xP(x)$	$x^2P(x)$
Head	1,000,002	.50	500,001	5×10^{11}
Tail	-1,000,000	.50	-500,000	5×10^{11}
			$\sum xP(x) = 1.00$	$\sum x^2P(x) = 10^{12}$

$$\mu = \sum xP(x) = \$1.00$$

$$\sigma = \sqrt{\sum x^2P(x) - \mu^2} = \sqrt{10^{12} - (1)^2} = \$1{,}000{,}000$$

Game I is preferable to Game II because the mean for Game I is greater than the mean for Game II. Although the mean for Game III is the same as Game I, the standard deviation for Game III is extremely high, making it very unattractive to a risk-adverse person. Thus, for most people, Game I is the best and, probably, Game III is the worst (due to its very high standard deviation).

5.125 Let x_1 = the number of contacts on the first day and x_2 = the number of contacts on the second day. The following table, which may be constructed with the help of a tree diagram, lists the various combinations of contacts during the two days and their probabilities. Note that the probability of each combination is obtained by multiplying the probabilities of the two events included in that combination since the events are independent.

x_1, x_2	Probability	y
(1, 1)	.0144	2
(1, 2)	.0300	3
(1, 3)	.0672	4
(1, 4)	.0084	5
(2, 1)	.0300	3
(2, 2)	.0625	4
(2, 3)	.1400	5
(2. 4)	.0175	6
(3, 1)	.0672	4
(3, 2)	.1400	5
(3, 3)	.3136	6
(3, 4)	.0392	7
(4, 1)	.0084	5
(4, 2)	.0175	6
(4, 3)	.0392	7
(4, 4)	.0049	8

Then the probability distribution of y is given in the following table.

y	$P(y)$
2	.0144
3	.0600
4	.1969
5	.2968
6	.3486
7	.0784
8	.0049

5.126 a. $\lambda = 5$ calls per 15-minute period

Using the table of Poisson probabilities:

$P(x \geq 9) = P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12) + P(x = 13) + P(x = 14) + P(x = 15)$

$= .0363 + .0181 + .0082 + .0034 + .0013 + .0005 + .0002 = .0680$

b. $\lambda = 7.5$ calls per 15-minute period

$P(x \geq 9) = P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12) + P(x = 13) + P(x = 14) + P(x = 15)$
$+ P(x = 16) + P(x = 17) + P(x = 18) + P(x = 19) + P(x = 20) = .1144 + .0858 + .0585 + .0366$
$+ .0211 + .0113 + 0057 + .0026 + .0012 + .0005 + .0002 + .0001 = .3380$

c. There is a 6.8% chance of observing 9 (or more) calls in the 15-minute period if the rate is actually 20 per hour and a 33.8% chance of observing 9 or more calls in a 15-minute period if the rate is actually 30 per hour. Therefore, the true rate is more likely to be 30 calls per hour.

d. You should advise the manager to hire a second operator since the rate of incoming calls is likely to be higher than 20 calls per hour.

5.127 There are a total of 27 outcomes for the game which can be determined utilizing a tree diagram. Three outcomes are favorable to Player A, 18 outcomes are favorable to Player B, and 6 outcomes are favorable to Player C. Player B's expected winnings are $\sum xP(x) = (0)(9/27) + (1)(18/27) = .67$ or 67¢. Player C's expected winnings are also 67¢: $\sum xP(x) = (0)(21/27) + (3)(6/27) = .67$. Since Player A has a probability of winning of 3/27, this player should be paid \$6 for winning so that $\sum xP(x) = (0)(24/27) + (6)(3/27) = .67$ or 67¢.

5.128 $\lambda = 10$ customers per hour

a. Let x_1 = the number of customers in the first hour and x_2 = the number of customers in the second hour. Since the situation is modeled using a Poisson random variable, arrivals are independent and the probabilities are calculated as follows:

$P(\text{exactly 4 customers in two hours}) = P(x_1 = 0, x_2 = 4) + P(x_1 = 1, x_2 = 3) + P(x_1 = 2, x_2 = 2)$
$+ P(x_1 = 3, x_2 = 1) + P(x_1 = 4, x_2 = 0) = (.0000)(.0189) + (.0005)(.0076) + (.0023)(.0023)$
$+ (.0076)(.0005) + (.0189)(.0000) = .00001289 \approx .0000$

b. Let x = the number of customers arriving in two hours. Since the average rate per hour is 10 customers, the average for two hours is $\lambda = (2)(10) = 20$ customers.

$$P(\text{exactly 4 customers in two hours}) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{(20)^4 e^{-20}}{4!} \approx .0000$$

5.129 a. .0211285 b. .047539 c. .4225690

Self-Review Test

1. See solution to Exercise 5.1.

2. The probability distribution table.

3. a **4.** b **5.** a

6. See solutions to Exercise 5.49 and Exercise 5.51b.

7. b **8.** a **9.** b **10.** a **11.** c

12. A hypergeometric probability distribution is used to find probabilities for the number of successes in a fixed number of trials, when the trials are not independent (such as sampling without replacement from a finite population.) Example 5-23 is an example of a hypergeometric probability distribution.

13. a

14. See solution to Exercise 5.79

15.

x	$P(x)$	$xP(x)$	x^2	$x^2P(x)$
0	.15	.00	0	.00
1	.24	.24	1	.24
2	.29	.58	4	1.16
3	.14	.42	9	1.26
4	.10	.40	16	1.60
5	.08	.40	25	2.00
		$\sum xP(x) = 2.04$		$\sum x^2P(x) = 6.26$

$\mu = \sum xP(x) = 2.04$ homes

$\sigma = \sqrt{\sum x^2 P(x) - \mu^2} = \sqrt{6.26 - (2.04)^2} = 1.449$ homes

The four real estate agents sell an average of 2.04 homes per week.

16. $n = 12$ and $p = .60$

a. i. $P(\text{exactly } 8) = P(x = 8) = {}_nC_x p^x q^{n-x} = {}_{12}C_8(.60)^8(.40)^4 = (495)(.01679616)(.0256) = .2128$

ii. $P(\text{at least } 6) = P(x \geq 6) = P(x = 6) + P(x = 7) + P(x = 8) + P(x = 9) + P(x = 10) + P(x = 11)$

$+ P(x = 12) = .1766 + .2270 + .2128 + .1419 + .0639 + .0174 + .0022 = .8418$

iii. $P(\text{less than } 4) = P(x < 4) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$

$= .0000 + .0003 + .0025 + .0125 = .0153$

b.

x	$P(x)$
0	.0000
1	.0003
2	.0025
3	.0125
4	.0420
5	.1009
6	.1766
7	.2270
8	.2128
9	.1419
10	.0639
11	.0174
12	.0022

$\mu = np = 12(.60) = 7.2$ adults

$\sigma = \sqrt{npq} = \sqrt{12(.60)(.40)} = 1.697$ adults

17. Let x denote the number of females in a sample of 4 volunteers from the 12 nominees. Then x is a hypergeometric random variable with: $N = 12$, $r = 8$, $N - r = 4$ and $n = 4$.

a. $P(\text{exactly } 3) = P(x = 3) = \dfrac{{}_rC_x \ {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_8C_3 \ {}_4C_1}{{}_{12}C_4} = \dfrac{(56)(4)}{495} = .4525$

b. $P(\text{exactly } 1) = P(x = 1) = \dfrac{{}_rC_x \ {}_{N-r}C_{n-x}}{{}_NC_n} = \dfrac{{}_8C_1 \ {}_4C_3}{{}_{12}C_4} = \dfrac{(8)(4)}{495} = .0646$

c. $P(\text{at most one}) = P(x \le 1) = P(x = 0) + P(x = 1)$

$= \dfrac{{}_8C_0 \ {}_4C_4}{{}_{12}C_4} + .0646 = \dfrac{(1)(1)}{495} + .0646 = .0020 + .0646 = .0666$

18. $\lambda = 10$ red light runners are caught per day.

Let x = number of drivers caught during rush hour on a given weekday.

a. i. $P(x = 14) = \dfrac{\lambda^x e^{-\lambda}}{x!} = \dfrac{(10)^{14} e^{-10}}{14!} = \dfrac{(100000000000000)(.0000453999)}{87{,}178{,}291{,}200} = .0521$

ii. $P(\text{at most } 7) = P(x \le 7) = P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$

$+ P(x = 6) + P(x = 7) = .0000 + .0005 + .0023 + .0076 + .0189 + .0378 + .0631 + .0901 = .2203$

iii. $P(13 \text{ to } 18) = P(13 \le x \le 18) = P(x = 13) + P(x = 14) + P(x = 15) + P(x = 16) + P(x = 17)$

$+ P(x = 18) = .0729 + .0521 + .0347 + .0217 + .0128 + .0071 = .2013$

b.

x	$P(x)$
0	.0000
1	.0005
2	.0023
3	.0076
4	.0189
5	.0378
6	.0631
7	.0901
8	.1126
9	.1251
10	.1251
11	.1137
12	.0948
13	.0729
14	.0521
15	.0347
16	.0217
17	.0128
18	.0071
19	.0037
20	.0019
21	.0009
22	.0004
23	.0002
24	.0001

19. See solution to Exercise 5.57.