Chapter Four

Section 4.1

4.1 An **experiment** is a process that, when performed, results in one and only one of many observations. An **outcome** is the result of the performance of an experiment. The collection of all outcomes for an experiment is called a **sample space**. A **simple event** is an event that includes one and only one of the final outcomes of an experiment. A **compound event** is a collection of more than one outcome of an experiment.

4.2 a. $S = \{1, 2, 3, 4, 5, 6\}$

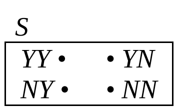
b. $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

c. $S = \{H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$

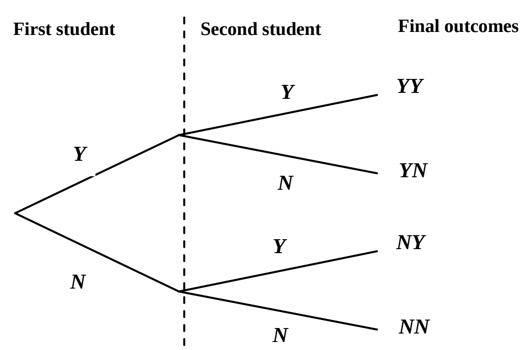
4.3 The experiment of selecting two items from the box without replacement has the following six possible outcomes: *AB*, *AC*, *BA*, *BC*, *CA*, *CB*. The sample space is written as $S = \{AB, AC, BA, BC, CA, CB\}$.

4.4 Let *Y* = student selected suffers from math anxiety and *N* = student selected does not suffer from math anxiety. The experiment of selecting two students has four outcomes: *YY, YN, NY,* and *NN.*

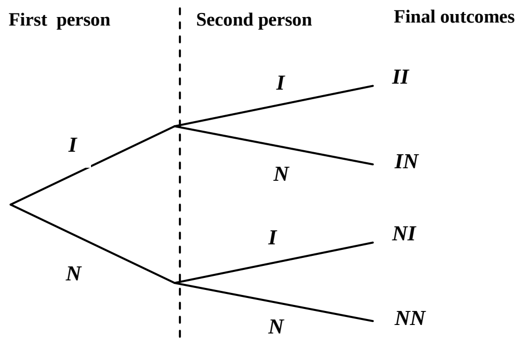
Venn Diagram



Tree Diagram

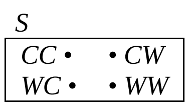


4.5 Let *I* = person has an iPod and *N* = person does not have an iPod. The experiment has four outcomes: *II, IN, NI,* and *NN.*

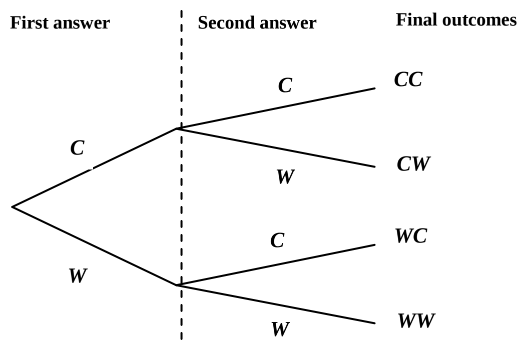


4.6 Let C = the answer selected is correct and W = the answer selected is wrong. This experiment has four outcomes: *CC*, *CW*, *WC*, and *WW*.

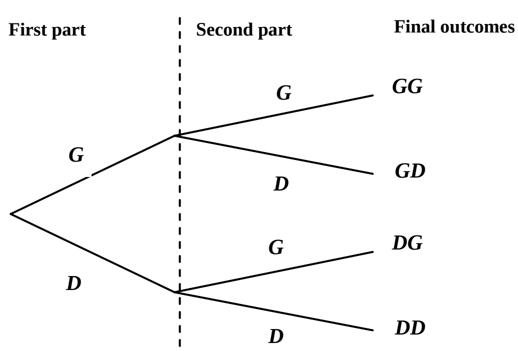
Venn Diagram



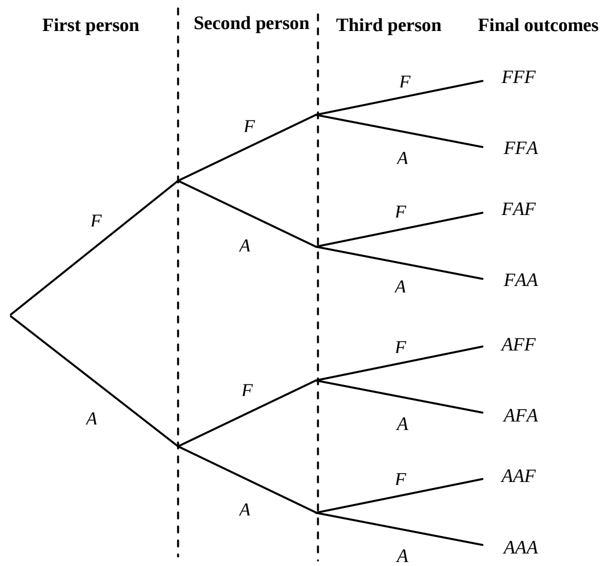
Tree Diagram



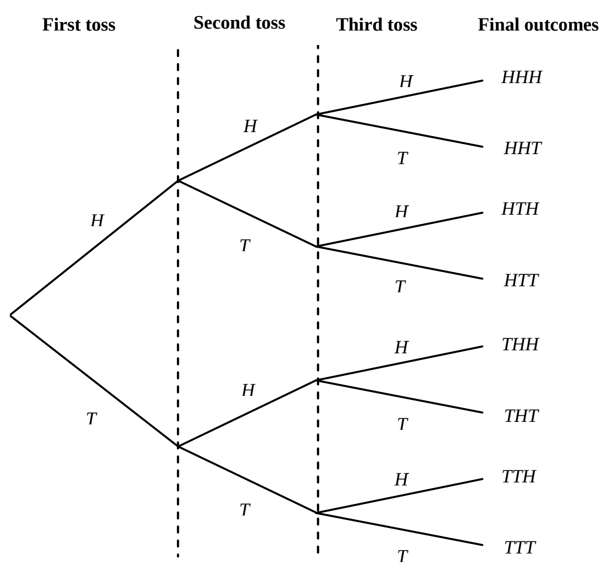
4.7 Let G = the selected part is good and D = the selected part is defective. The four outcomes for this experiment are: *GG*, *GD*, *DG*, and *DD*.



4.8 Let F = person selected is in favor of tax increase and A = person selected is against tax increase. The experiment of selecting three persons has eight outcomes: *FFF*, *FFA*, *FAF*, *FAA*, *AFF*, *AFA*, *AAF*, and *AAA*. The sample space is written as $S = \{FFF, FFA, FAF, FAA, AFF, AFA, AAF, AAA\}$.



4.9 Let H = a toss results in a head and T = a toss results in a tail. The sample space is written as S = {*HHH, HHT, HTH, HTT, THH, THT, TTH, TTT*}.



4.10
a. {*YY*}; a simple event
b. {*YN*, *NY*}; a compound event
c. {*NY*}; a simple event
d. {*NN*}; a simple event

4.11
a. {*IN*, *NI*}; a compound event
b. {*II*, *IN*, *NI*}; a compound event
c. {*NN*, *NI*, *IN*}; a compound event
d. {*IN*}; a simple event

4.12
a. {*CC*}; a simple event
c. {*CW*}; a simple event

b. $\{CC, CW, WC\}$; a compound event

d. $\{CW, WC\}$; a compound event

4.13 a. $\{DG, GD, GG\}$; a compound event

b. $\{DG, GD\}$; a compound event

c. $\{GD\}$; a simple event

d. $\{DD, DG, GD\}$; a compound event

4.14 a. $\{FFF, FFA, FAF, AFF\}$; a compound event

b. $\{FFA, FAF, AFF\}$; a compound event

c. $\{FFA, FAF, FAA, AFF, AFA, AAF, AAA\}$; a compound event

d. $\{FAA, AFA, AAF, AAA\}$; a compound event

Section 4.2

4.15 1. The probability of an event always lies in the range zero to 1, that is:

$$0 \le P(E_i) \le 1 \text{ and } 0 \le P(A) \le 1$$

2. The sum of the probabilities of all simple events for an experiment is always 1, that is:

$$\sum P(E_i) = P(E_1) + P(E_2) + P(E_3) + \cdots = 1$$

4.16 An event that cannot occur is called an **impossible event**. The probability that such an event will occur is zero. An event that is certain to occur is called a **sure event**. The probability that this event will occur is 1.

4.17 1. **Classical probability approach:** When all outcomes are equally likely, the probability of an event *A* is given by: $P(A) = \dfrac{\text{Number of outcomes favorable to A}}{\text{Total number of outcomes for the experiment}}$

For example, the probability of observing a head when a fair coin is tossed once is 1/2.

2. **Relative frequency approach:** If an event *A* occurs *f* times in *n* repetitions of an experiment, then $P(A)$ is approximately f/n. As the experiment is repeated more and more times, f/n approaches $P(A)$. For example, if 50 of the last 5000 cars off the assembly line are lemons, the probability that the next car is a lemon is approximately $P(\text{lemon}) = f/n = 50/5000 = .01$.

3. **Subjective probability approach:** Probabilities are assigned based on subjective judgment, experience, information and belief. For example, a teacher might estimate the probability of a student earning an A on a statistics test to be 1/6 based on previous classes.

4.18 The classical approach is used when all the outcomes of an experiment are equally likely. The relative frequency approach is used when all the outcomes are not equally likely, but the experiment can be performed repeatedly to generate data.

4.19 The values $-.55$, 1.56, 5/3, and $-2/7$ cannot be probabilities of events because the probability of an event can never be less than zero or greater than one.

4.20 The values −.09, 1.42, 9/4, and −1/4 cannot be probabilities of events because the probability of an event can never be less than zero or greater than one.

4.21 These two outcomes would not be equally likely unless exactly half of the passengers entering the metal detectors set it off, which is unlikely. We would have to obtain a random sample of passengers going through New York's JFK airport, collect information on whether they set off the metal detector or not, and use the relative frequency approach to find the probabilities.

4.22 We would use the classical approach, since each of the 32 applicants is equally likely to be selected. Thus, the probability of selecting an experienced candidate is 7/32 and that of selecting an inexperienced candidate is 25/32.

4.23 This is a case of subjective probability because the given probability is based on the president's judgment.

4.24 This is a case of subjective probability because the given probability is based on the coach's belief.

4.25 a. P(marble selected is red) = 18/40 = .45
b. P(marble selected is green) = 22/40 = .55

4.26 a. P(a number less than 5 is obtained) = 4/6 = .6667
b. P(a number 3 to 6 is obtained) = 4/6 = .6667

4.27 P(adult selected has shopped on the internet) = 1320/2000 = .66

4.28 P(student selected has volunteered before) = 28/42 = .6667

4.29 P(car owner selected owns a hybrid car) = 8/50 = .16

4.30 Number of families who did pay income tax last year = 3000 − 600 = 2400
P(family selected did pay income tax last year) = 2400/3000 = .800

4.31 a. P(her answer is correct) = 1/5 = .2
b. P(her answer is wrong) = 4/5 = .8
Yes, these probabilities add up to 1.0 because this experiment has two and only two outcomes, and according to the second property of probability, the sum of their probabilities must be equal to 1.0.

4.32 a. P(eligible voter is registered) = 972/1265 = .768
b. Number of eligible voters not registered = 1265 − 972 = 293
P(eligible voter is not registered) = 293/1265 = .232
Yes, the sum of these probabilities is 1.0 because of the second property of probability.

4.33 P(person selected is a woman) = 4/6 = .6667

P(person selected is a man) = 2/6 = .3333

Yes, the sum of these probabilities is 1.0 because of the second property of probability.

4.34 P(company selected offers free psychiatric help) = 120/500 = .240

Number of companies that do not offer free psychiatric help = 500 − 120 = 380

P(company selected does not offer free psychiatric help) = 380/500 = .760

Yes, these probabilities add up to 1.0 because of the second property of probability.

4.35 P(company selected offers free health fitness center on the company premises) = 130/400 = .325

Number of companies that do not offer free health fitness center on the company premises = 400 − 130 = 270

P(company selected does not offer free health fitness center on the company premises) = 270/400 = .675

Yes, the sum of the probabilities is 1.0 because of the second property of probability.

4.36 a. P(company closed down or moved) = 7400/15000 = .4933

b. P(insufficient work) = 4600/15000 = .3067

c. Number of employees losing jobs because position abolished = 15000 − 7400 − 4600 = 3000

P(position abolished) = 3000/15000 = .2000

Yes, the sum of these three probabilities is 1.0 because of the second property of probability.

4.37

Credit Cards	Frequency	Relative Frequency
0	80	.098
1	116	.141
2	94	.115
3	77	.094
4	43	.052
5 or more	410	.500

a. P(person selected has three credit cards) = .094

b. P(person selected has five or more cards) = .500

4.38

Income	Frequency	Relative Frequency
Less than $40,000	70	.140
$40,000 to $80,000	220	.440
More than $80,000	210	.420

a. P(income is less than \$40,000) = .140

b. P(income is more than \$80,000) = .420

4.39 Take a random sample of families from Los Angeles and determine how many of them earn more than $175,000 per year. Then use the relative frequency approach.

4.40 Roll the die repeatedly a large number of times, recording how many times each of the six outcomes occurs. Then apply the relative frequency approach.

<u>Sections 4.3 - 4.7</u>

4.41 **Marginal probability** is the probability of a single event without consideration of any other event. **Conditional probability** is the probability that an event will occur given that another event has already occurred. For example, when a single die is rolled, the marginal probability of a number less than 4 is 1/2; the conditional probability of an odd number given that a number less than 4 has occurred is 2/3.

4.42 Events that cannot occur together are called **mutually exclusive** events. For example, if a student is randomly selected, the events "male" and "female" are mutually exclusive events. The events "male" and "senior" are mutually nonexclusive events.

4.43 Two events are **independent** if the occurrence of one does not affect the probability of the occurrence of the other. Two events are **dependent** if the occurrence of one affects the probability of the occurrence of the other. If two events A and B satisfy the condition $P(A|B) = P(A)$, or $P(B|A) = P(B)$, they are independent; otherwise they are dependent.

4.44 The **complement** of event A consists of all the outcomes for an experiment that are not included in A. The sum of the probabilities of two complementary events is 1.

4.45 Total outcomes for four rolls of a die = $6 \times 6 \times 6 \times 6 = 1296$

4.46 Total outcomes for 10 tosses of a coin = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1024$

4.47
a. Events A and B are not mutually exclusive since they have the element "2" in common.
b. $P(A) = 3/8$ and $P(A|B) = 1/3$. Since these probabilities are not equal, A and B are dependent.
c. $\overline{A} = \{1, 3, 4, 6, 8\}$; $P(\overline{A}) = 5/8 = .625$

$\overline{B} = \{1, 3, 5, 6, 7\}$; $P(\overline{B}) = 5/8 = .625$

4.48
a. Events A and B are mutually exclusive since they have no common element.
b. $P(A) = 4/10$ and $P(A|B) = 0$. Since these two probabilities are not equal, A and B are dependent.
c. $\overline{A} = \{1, 2, 5, 7, 8, 10\}$; $P(\overline{A}) = 6/10 = .600$

$\overline{B} = \{3, 4, 6, 7, 8, 9, 10\}$; $P(\overline{B}) = 7/10 = .700$

4.49 Total selections = $10 \times 5 = 50$

4.50 Total selections = $4 \times 8 \times 12 = 384$

4.51 Total outcomes = 4 × 8 × 5 × 6 = 960

4.52 Total outcomes = 8 × 6 × 5 = 240

4.53 a. i. *P*(selected adult has never shopped on the internet) = 1200/2000 = .600
ii. *P*(selected adult is a male) = 1200/2000 = .600
iii. *P*(selected adult has shopped on the internet given that this adult is a female) = 300/800 = .375
iv. *P*(selected adult is a male given that this adult has never shopped on the internet) = 700/1200 = .583

b. The events "male" and "female" are mutually exclusive because they cannot occur together. The events "have shopped" and "male" are not mutually exclusive because they can occur together.

c. *P*(female) = 800/2000 = .400 and *P*(female | have shopped) = 300/800 = .375. Since these probabilities are not equal, the events "female" and "have shopped" are dependent.

4.54 a. i. *P*(does not favor) = 373/1012 = .369
ii. *P*(Republican/Republican leaner) = 509/1012 = .503
iii. *P*(favors | Republican/Republican leaner) = 381/509 = .749
iv. *P*(Republican/Republican leaner | does not favor) = 128/373 = .343

b. The events "favors" and "does not favor" are mutually exclusive because they cannot occur together. The events "does not favor" and "Republican/Republican leaner" are not mutually exclusive because they can occur together.

c. *P*(does not favor) = 373/1012 = .369 and *P*(does not favor | Republican/Republican leaner) = 128/509 = .251. Since these two probabilities are not equal, the events "does not favor" and "Republican/Republican leaner" are dependent.

4.55 a. i. *P*(in favor) = 695/2000 = .3475
ii. *P*(against) = 1085/2000 = .5425
iii. *P*(in favor | female) = 300/1100 = .2727
iv. *P*(male | no opinion) = 100/220 = .4545

b. The events "male" and "in favor" are not mutually exclusive because they can occur together. The events "in favor" and "against" are mutually exclusive because they cannot occur together.

c. *P*(female) = 1100/2000 = .5500 and *P*(female | no opinion) = 120/220 = .5455. Since these two probabilities are not equal, the events "female" and "no opinion" are dependent.

4.56 a. i. *P*(woman) = 200/500 = .400
ii. *P*(has retirement benefits) = 375/500 = .750
iii. *P*(has retirement benefits | man) = 225/300 = .750
iv. *P*(woman | does not have retirement benefits) = 50/125 = .400

b. The events "man" and "yes" are not mutually exclusive because they can occur together. The events "yes" and "no" are mutually exclusive because they cannot occur together.

c. *P*(woman) = 200/500 = .400 and *P*(woman | yes) = 150/375 = .400. Since these two probabilities are equal, the events "woman" and "yes" are independent.

4.57 a. i. *P*(more than 1 hour late) = 172/1700 = .1012

ii. *P*(less than 30 minutes late) = 822/1700 = .4835

iii. *P*(Airline A's flight | 30 minutes to 1 hour late) = 390/706 = .5524

iv. *P*(more than 1 hour late | Airline B's flight) = 80/789 = .1014

b. The events "Airline A" and "more than 1 hour late" are not mutually exclusive because they can occur together. The events "less than 30 minutes late" and "more than 1 hour late" are mutually exclusive because they cannot occur together.

c. *P*(Airline B) = 789/1700 = .4641 and *P*(Airline B | 30 minutes to 1 hour late) = 316/706 = .4476. Since these two probabilities are not equal, the events "Airline B" and "30 minutes to 1 hour late" are dependent.

4.58 a. i. *P*(better off) = 1010/2000 = .5050

ii. *P*(better off | less than high school) = 140/400 = .3500

iii. *P*(worse off | high school) = 300/1000 = .3000

iv. *P*(the same | more than high school) = 110/600 = .1833

b. The events "better off" and "high school education" are not mutually exclusive because they can occur together. The events "less than high school" and "more than high school" are mutually exclusive because they cannot occur together.

c. *P*(worse off) = 570/2000 = .2850 and *P*(worse off | more than high school) = 70/600 = .1167. Since these two probabilities are not equal, the events "worse off" and "more than high school" are dependent.

4.59 *P*(pediatrician) = 25/160 = .1563 and *P*(pediatrician | female) = 20/75 = .2667. Since these two probabilities are not equal, the events "female" and "pediatrician" are dependent. The events are not mutually exclusive because they can occur together.

4.60 Let D = the CD selected is defective and F = the CD selected is made on Machine I. Then, $P(D)$ = 20/100 = .2000 and $P(D|F)$ = 10/60 = .1667. Since these two probabilities are not equal, the events "machine type" and "defective CD" are not independent.

4.61 *P*(business major) = 11/30 = .3667 and *P*(business major | female) = 9/16 = .5625. Since these two probabilities are not equal, the events "female" and "business major" are not independent. The events are not mutually exclusive because they can occur together.

4.62 The experiment involving two tosses of a coin has four outcomes: *HH, HT, TH,* and *TT,* where *H* denotes the event that a head is obtained and *T* that a tail is obtained on any toss. Events *A* and *B* contain the following outcomes: $A = \{HH, HT, TH\}$ and $B = \{TT\}$.

a. Since events *A* and *B* do not contain any common outcome, they are mutually exclusive events. $P(A) = 3/4 = .750$ and $P(A|B) = 0$. Since these two probabilities are not equal, *A* and *B* are not independent events.

b. Events *A* and *B* are complementary events because they do not contain any common outcome and, taken together, they contain all the outcomes for this experiment.
$P(B) = 1/4 = .250$ and $P(A) = 1 – P(B) = 1 – .250 = .750$

4.63 Event *A* will occur if either a 1-spot or a 2-spot is obtained on the die. Thus, $P(A) = 2/6 = .3333$. The complementary event of *A* is that a 3-spot, or a 4-spot, or a 5-spot, or a 6-spot is obtained on the die. Hence, $P(\bar{A}) = 1 – .3333 = .6667$.

4.64 The two complementary events are that the child selected lived with both of their parents in the same household and that the child selected lived with at most one parent in the household.
P(both) = 52.1/73.7 = .7069 and *P*(at most one parent) = 21.6/73.7 = .2931

4.65 The complementary event is that the college student attended no major league baseball games last year. The probability of this complementary event is 1 – .12 = .88

Section 4.8

4.66 The **intersection** of two events is the collection of all the outcomes that are common to both events. For example, if $A = \{1, 2, 3\}$ and $B = \{1, 3, 5\}$, then the intersection of *A* and *B* is the event $\{1, 3\}$.

4.67 The **joint probability** of two events is the probability of the intersection of the two events. For example, suppose a die is rolled once. Let *A* = a number greater than 4 occurs = $\{5, 6\}$ and *B* = an odd number occurs = $\{1, 3, 5\}$. Then $P(A \text{ and } B) = P(\{5\}) = 1/6$ is the joint probability of *A* and *B*.

4.68 Unlike the rule for independent events, the rule for dependent events requires a conditional probability. Thus, if *A* and *B* are dependent, then $P(A \text{ and } B) = P(A)P(B|A)$. If *A* and *B* are independent events, then $P(A \text{ and } B) = P(A)P(B)$.

4.69 The joint probability of two mutually exclusive events is zero. For example, consider one roll of a die. Let *A* = a number less than 4 occurs = $\{1, 2, 3\}$ and *B* = a number greater than 3 occurs = $\{4, 5, 6\}$. Then, *A* and *B* are mutually exclusive events, since they have no outcomes in common. The event (*A* and *B*) is impossible, and so $P(A \text{ and } B) = 0$.

4.70 a. $P(A \text{ and } B) = P(A)P(B|A) = (.40)(.25) = .100$

b. $P(A \text{ and } B) = P(B \text{ and } A) = P(B)P(A|B) = (.65)(.36) = .234$

4.71 a. $P(A \text{ and } B) = P(B \text{ and } A) = P(B)P(A|B) = (.59)(.77) = .4543$

b. $P(A \text{ and } B) = P(A)P(B|A) = (.28)(.35) = .0980$

4.72 a. $P(A \text{ and } B) = P(A)P(B) = (.61)(.27) = .1647$

b. $P(A \text{ and } B) = P(A)P(B) = (.39)(.63) = .2457$

4.73 a. $P(A \text{ and } B) = P(A)P(B) = (.20)(.76) = .1520$

b. $P(A \text{ and } B) = P(A)P(B) = (.57)(.32) = .1824$

4.74 a. $P(A \text{ and } B \text{ and } C) = P(A)P(B)P(C) = (.20)(.46)(.25) = .0230$

b. $P(A \text{ and } B \text{ and } C) = P(A)P(B)P(C) = (.44)(.27)(.43) = .0511$

4.75 a. $P(A \text{ and } B \text{ and } C) = P(A)P(B)P(C) = (.49)(.67)(.75) = .2462$

b. $P(A \text{ and } B \text{ and } C) = P(A)P(B)P(C) = (.71)(.34)(.45) = .1086$

4.76 $P(B|A) = P(A \text{ and } B) / P(A) = .24/.30 = .800$

4.77 $P(A|B) = P(A \text{ and } B) / P(B) = .45/.65 = .6923$

4.78 $P(B) = P(A \text{ and } B) / P(A|B) = .36/.40 = .900$

4.79 $P(A) = P(A \text{ and } B) / P(B|A) = .58/.80 = .725$

4.80 Let V = have been victimized and N = have never been victimized.

a. i. $P(V \text{ and } C) = P(V)P(C|V) = \left(\frac{312}{1800}\right)\left(\frac{61}{312}\right) = .0339$

ii. $P(N \text{ and } A) = P(N)P(A|N) = \left(\frac{1488}{1800}\right)\left(\frac{698}{1488}\right) = .3878$

b. $P(B \text{ and } C) = 0$ since B and C are mutually exclusive events.

4.81 Let M = male, F = female, G = graduated, and N = did not graduate.

a. i. $P(F \text{ and } G) = P(F)P(G|F) = \left(\frac{165}{346}\right)\left(\frac{133}{165}\right) = .3844$

ii. $P(M \text{ and } N) = P(M)P(N|M) = \left(\frac{181}{346}\right)\left(\frac{55}{181}\right) = .1590$

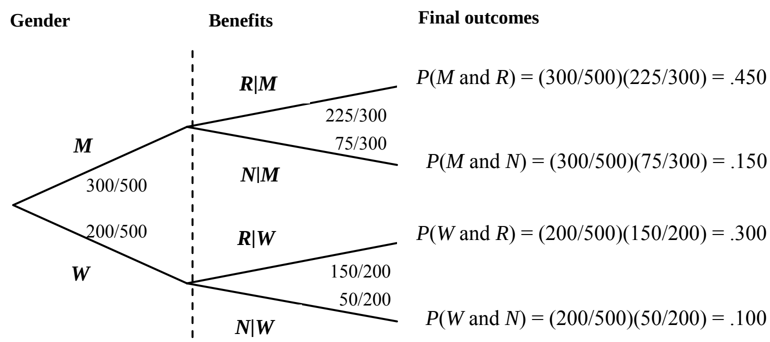
b. $P(G \text{ and } N) = 0$ since G and N are mutually exclusive events.

4.82 Let M = man, W = woman, R = has retirement benefits, and N = does not have retirement benefits.

a. i. $P(W \text{ and } R) = P(W)P(R|W) = \left(\frac{200}{500}\right)\left(\frac{150}{200}\right) = .300$

ii. $P(N \text{ and } M) = P(N)P(M|N) = \left(\frac{125}{500}\right)\left(\frac{75}{125}\right) = .150$

b.

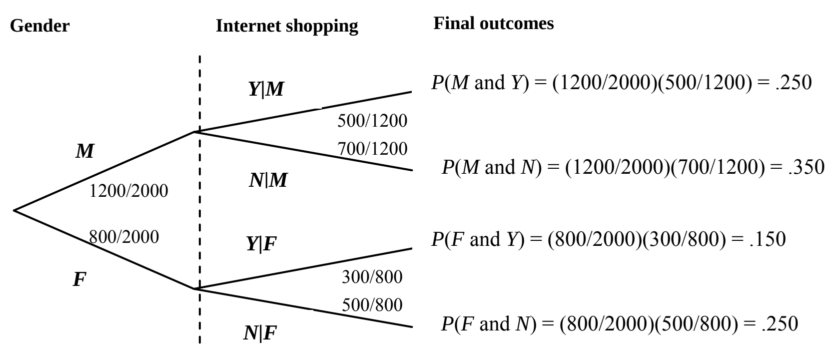


4.83 Let M = male, F = female, Y = has shopped at least once on the internet, and N = has never shopped on the internet.

a. i. $P(N \text{ and } M) = P(N)P(M|N) = \left(\frac{1200}{2000}\right)\left(\frac{700}{1200}\right) = .350$

ii. $P(Y \text{ and } F) = P(Y)P(F|Y) = \left(\frac{800}{2000}\right)\left(\frac{300}{800}\right) = .150$

b.



4.84 a. i. P(more than 1 hour late and on Airline A)

$= P(\text{more than 1 hour late})\, P(\text{on Airline A} \mid \text{more than 1 hour late}) = \left(\frac{172}{1700}\right)\left(\frac{92}{172}\right) = .0541$

ii. P(on Airline B and less than 30 minutes late)

$= P(\text{on Airline B})\, P(\text{less than 30 minutes late} \mid \text{on Airline B}) = \left(\frac{789}{1700}\right)\left(\frac{393}{789}\right) = .2312$

b. P(30 minutes to 1 hour late and more than 1 hour late) = 0 since "30 minutes to 1 hour late" and "more than 1 hour late" are mutually exclusive events.

4.85 a. i. P(better off and high school)

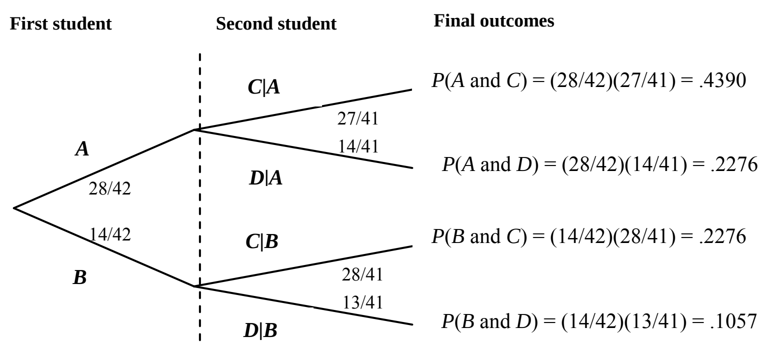
$$= P(\text{better off})\ P(\text{high school} \mid \text{better off}) = \left(\frac{1010}{2000}\right)\left(\frac{450}{1010}\right) = .225$$

ii. P(more than high school and worse off)

$$= P(\text{more than high school})\ P(\text{worse off} \mid \text{more than high school}) = \left(\frac{600}{2000}\right)\left(\frac{70}{600}\right) = .035$$

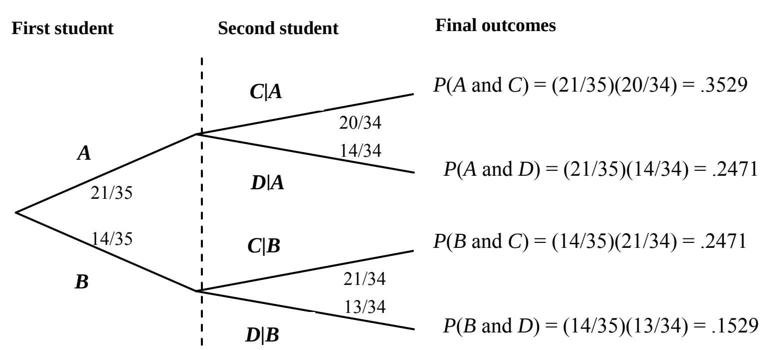
b. P(worse off and better off) = 0 because "worse off" and "better off" are mutually exclusive events.

4.86 Let A = first student selected has volunteered before, B = first student selected has not volunteered before, C = second student selected has volunteered before, and D = second student selected has not volunteered before.



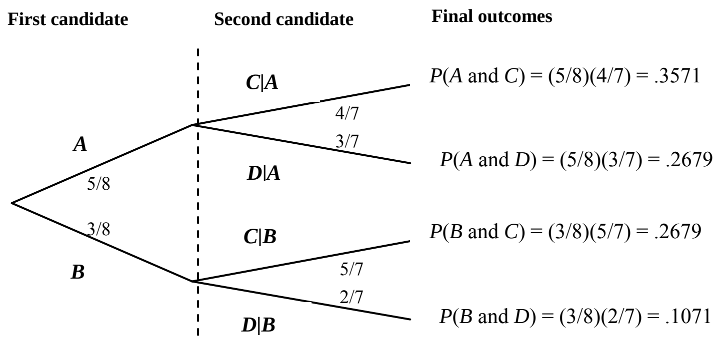
Then, P(both students have volunteered before) = $P(A \text{ and } C) = P(A)P(C\,|A)$ = .4390.

4.87 Let A = first selected student favors abolishing the Electoral College, B = first selected student favors keeping the Electoral College, C = second selected student favors abolishing the Electoral College, and D = second selected student favors keeping the Electoral College.



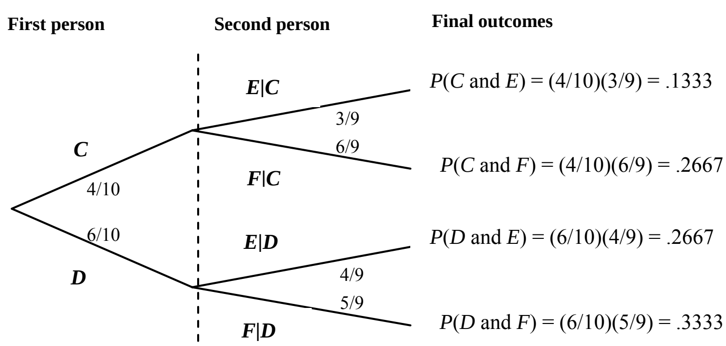
Then, P(both student favors abolishing the Electoral College) = $P(A \text{ and } C) = P(A)P(C|A)$ = .3529.

4.88 Let A = first candidate selected is a woman, B = first candidate selected is a man, C = second candidate selected is a woman, and D = second candidate selected is a man.



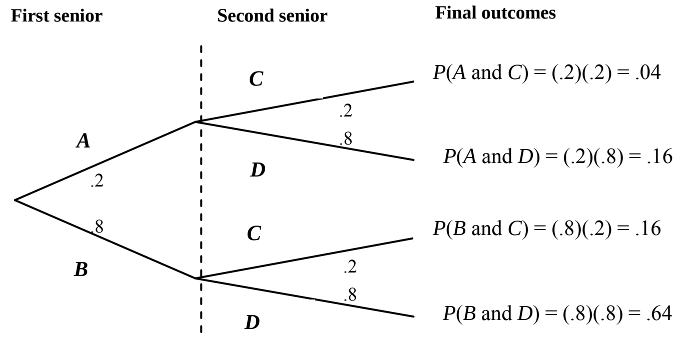
Then P(both candidates selected are women) = $P(A \text{ and } C) = P(A)P(C|A) = .3571$.

4.89 Let C = first selected person has a type A personality, D = first selected person has a type B personality, E = second selected person has a type A personality, and F = second selected person has a type B personality.



Then P(the first person has a type A personality and the second has a type B personality) = $P(C \text{ and } F)$ = $P(C)P(F|C) = .2667$.

4.90 Let A = first selected senior has spent Spring Break in Florida, B = first selected senior has never spent Spring Break in Florida, C = second selected senior has spent Spring Break in Florida, and D = second selected senior has never spent Spring Break in Florida.



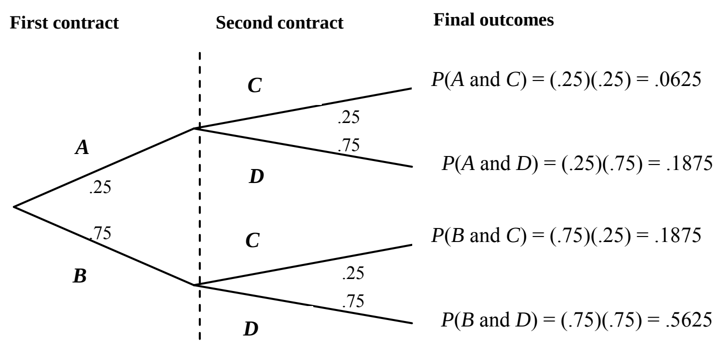
Because students are independent, P(first senior has never gone to Florida for Spring Break and second senior has gone to Florida for Spring Break) = $P(B \text{ and } C) = P(B)P(C) = .160$.

4.91 Let A = first student selected has student loans to pay off, B = first student selected does not have student loans to pay off, C = second student selected has student loans to pay off, and D = second student selected does not have student loans to pay off. Because students are independent, P(neither student selected has loans to pay off) = $P(B \text{ and } D) = P(B)P(D) = (.4)(.4) = .16$

4.92 Let A = wins first contract, B = does not win first contract, C = wins second contract, and D = does not win second contract.

a. $P(A \text{ and } C) = P(A)P(C) = (.25)(.25) = .0625$

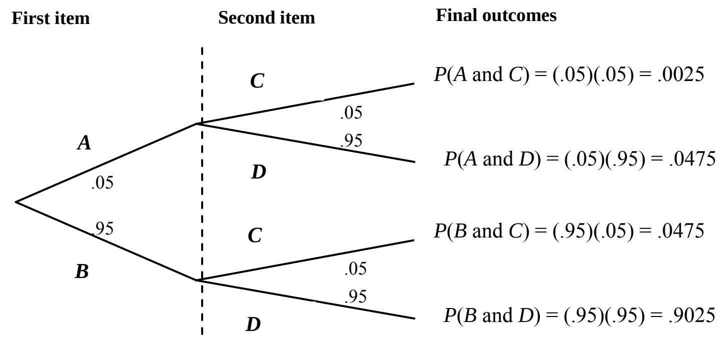
b. $P(B \text{ and } D) = P(B)P(D) = (.75)(.75) = .5625$



4.93 Let A = first item is returned, B = first item is not returned, C = second item is returned, and D = second item is not returned.

a. $P(A \text{ and } C) = P(A)P(C) = (.05)(.05) = .0025$

c. $P(B \text{ and } D) = P(B)P(D) = (.95)(.95) = .9025$



4.94 Let N_1 = first person selected is not allergic to the drug, N_2 = second person selected is not allergic to the drug, and N_3 = third person selected is not allergic to the drug. Then, $P(N_1 \text{ and } N_2 \text{ and } N_3) = P(N_1)P(N_2)\, P(N_3) = (.97)(.97)(.97) = .9127$.

4.95 Let D_1 = first farmer selected is in debt, D_2 = second farmer selected is in debt, and D_3 = third farmer selected is in debt. Then, $P(D_1 \text{ and } D_2 \text{ and } D_3) = P(D_1)P(D_2)\, P(D_3) = (.80)(.80)(.80) = .5120$.

4.96 Let L = student has loans to pay off and M = student is male. Since $P(L) = .60$ and $P(L \text{ and } M) = .24$, $P(M \mid L) = P(L \text{ and } M) / P(L) = .24/.60 = .40$.

4.97 Let F = employee selected is a female and M = employee selected is married. Since $P(F) = .36$ and $P(F \text{ and } M) = .19$, $P(M \mid F) = P(F \text{ and } M) / P(F) = .19/.36 = .5278$.

4.98 Let C = person selected cutting back somewhat and D = person selected is delaying the purchase of a new car. Then
$P(C \text{ and } D) = 97/629 = .1542$ and $P(C) = 322/629 = .5119$, and
$P(\text{D}|C) = P(C \text{ and } D) / P(C) = .1542/.5119 = .3012$

4.99 Let A = adult in small town lives alone and Y = adult in small town has at least one pet. Since $P(A) = .20$ and $P(A \text{ and } Y) = .08$, $P(Y|A) = P(A \text{ and } Y) / P(A) = .08/.20 = .400$

Section 4.9

4.100 The **union** of two events A and B is the event "A or B", which is the collection of all outcomes belonging to either A or B or both A and B. For example, if $A = \{1, 2, 3\}$ and $B = \{2, 4, 6\}$, then $(A \text{ or } B) = \{1, 2, 3, 4, 6\}$.

4.101 When two events are mutually exclusive, their joint probability is zero and is dropped from the formula. So, if A and B are mutually nonexclusive events, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. However, if A and B are mutually exclusive events, then $P(A \text{ or } B) = P(A) + P(B) - 0 = P(A) + P(B)$.

4.102 When we compute $P(A) + P(B)$ and events A and B are not mutually exclusive, $P(A \text{ and } B)$ is added in once with $P(A)$ and again with $P(B)$. Therefore, we need to subtract $P(A \text{ and } B)$ to avoid double counting the outcomes in the intersection. Consider Table 4.11 in the text. Let A = an even number is obtained on the first roll and B = the sum of the numbers obtained in two rolls is equal to 7. Then,

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{18}{36} + \frac{6}{36} - \frac{3}{36} = \frac{21}{36} = .5833$.

4.103 The formula $P(A \text{ or } B) = P(A) + P(B)$ is used when A and B are mutually exclusive events. For example, consider Table 4.11 in the text. Let A = an odd number on both rolls and B = the sum is an odd number. Since A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B) = \frac{9}{36} + \frac{18}{36} = \frac{27}{36} = .75$.

4.104 a. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .58 + .66 - .57 = .67$

b. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .72 + .42 - .39 = .75$

4.105 a. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .18 + .49 - .11 = .56$

b. $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = .73 + .71 - .68 = .76$

4.106 a. $P(A \text{ or } B) = P(A) + P(B) = .47 + .32 = .79$

b. $P(A \text{ or } B) = P(A) + P(B) = .16 + .59 = .75$

4.107 a. $P(A \text{ or } B) = P(A) + P(B) = .25 + .27 = .52$

b. $P(A \text{ or } B) = P(A) + P(B) = .58 + .09 = .67$

4.108 Let V = have been victimized and N = have never been victimized.

a. $P(V \text{ or } B) = P(V) + P(B) - P(V \text{ and } B) = \frac{312}{1800} + \frac{592}{1800} - \frac{145}{1800} = .4217$

b. $P(N \text{ or } C) = P(N) + P(C) - P(N \text{ and } C) = \frac{1488}{1800} + \frac{404}{1800} - \frac{343}{1800} = .8606$

4.109 Let M = basketball player selected is a male, F = basketball player selected is a female, G = player selected has graduated, and N = player selected has not graduated.

a. $P(F \text{ or } N) = P(F) + P(N) - P(F \text{ and } N) = \frac{165}{346} + \frac{87}{346} - \frac{32}{346} = .6358$

b. $P(G \text{ or } M) = P(G) + P(M) - P(G \text{ and } M) = \frac{259}{346} + \frac{181}{346} - \frac{126}{346} = .9075$

4.110 Let M = man, W = woman, Y = has retirement benefits, and N = does not have retirement benefits.

a. $P(W \text{ or } Y) = P(W) + P(Y) - P(W \text{ and } Y) = \frac{200}{500} + \frac{375}{500} - \frac{150}{500} = .850$

b. $P(N \text{ or } M) = P(N) + P(M) - P(N \text{ and } M) = \frac{125}{500} + \frac{300}{500} - \frac{75}{500} = .700$

4.111 Let M = male, F = female, Y = this adult has shopped on the internet, and N = this adult has never shopped on the internet.

a. $P(N \text{ or } F) = P(N) + P(F) - P(N \text{ and } F) = \frac{1200}{2000} + \frac{800}{2000} - \frac{500}{2000} = .750$

b. $P(M \text{ or } Y) = P(M) + P(Y) - P(M \text{ and } Y) = \frac{1200}{2000} + \frac{800}{2000} - \frac{500}{2000} = .750$

c. Since Y and N are mutually exclusive events, $P(Y \text{ or } N) = P(Y) + P(N) = \frac{800}{2000} + \frac{1200}{2000} = 1.0$.
In fact, these two events are complementary.

4.112 Let A = Airline A, B = Airline B, E = less than 30 minutes late, F = 30 minutes to 1 hour late, and G = more than 1 hour late.

a. $P(G \text{ or } A) = P(G) + P(A) - P(G \text{ and } A) = \frac{172}{1700} + \frac{911}{1700} - \frac{92}{1700} = .5829$

b. $P(B \text{ or } E) = P(B) + P(E) - P(B \text{ and } E) = \frac{789}{1700} + \frac{822}{1700} - \frac{393}{1700} = .7165$

c. Since A and B are mutually exclusive events, $P(A \text{ or } B) = P(A) + P(B) = \frac{911}{2000} + \frac{789}{2000} = 1.0$
In fact, these two events are complementary.

4.113 Let B = better off, S = same, W = worse off, L = less than high school, H = high school, and M = more than high school.

a. $P(B \text{ or } H) = P(B) + P(H) - P(B \text{ and } H) = \frac{1010}{2000} + \frac{1000}{2000} - \frac{450}{2000} = .780$

b. $P(M \text{ or } W) = P(M) + P(W) - P(M \text{ and } W) = \frac{600}{2000} + \frac{570}{2000} - \frac{70}{2000} = .550$

c. Since B and W are mutually exclusive events, $P(B \text{ or } W) = P(B) + P(W) = \frac{1010}{2000} + \frac{570}{2000} = .790$.

4.114 Let T = vehicle selected ticketed and V = vehicle selected vandalized. Then,
$P(T \text{ or } V) = P(T) + P(V) - P(T \text{ and } V) = .35 + .15 - .10 = .40$

4.115 Let W = family selected owns a washing machine and V = family selected owns a DVD player.
Then, $P(W \text{ or } V) = P(W) + P(V) - P(W \text{ and } V) = .68 + .81 - .58 = .91$

4.116 Let B = wedding day has bad weather and D = wedding day has a disruptive incident. Then,
$P(B \text{ or } D) = P(B) + P(D) - P(B \text{ and } D) = .25 + .15 - .08 = .32$

4.117 Let F = teacher selected is a female and S = teacher selected holds a second job. Then,
$P(F \text{ or } S) = P(F) + P(S) - P(F \text{ and } S) = .68 + .38 - .29 = .77$

4.118 Let M = person is currently married and N = person has never been married. Since these events are mutually exclusive, $P(M \text{ or } N) = P(M) + P(N) = \frac{123.7}{238} + \frac{71.5}{238} = .8202$.

This probability is not equal to 1.0 because there are people who have previously been married but are not currently married.

4.119 Let A = person owns home with three bedrooms, B = person owns home with four bedrooms. Since A and B are mutually exclusive, $P(A \text{ or } B) = P(A) + P(B) = \frac{800}{2000} + \frac{600}{2000} = .70$.

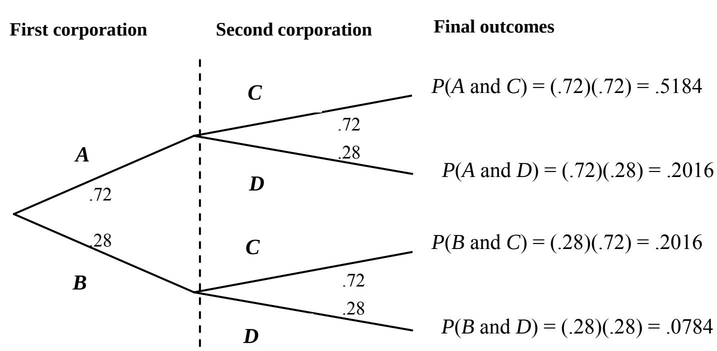
This probability is not equal to 1.0 because some people own homes with fewer than three bedrooms and some own homes with more than four bedrooms.

4.120 Let A = student selected will get an A in economics and B = student selected will get a B in economics. Since A and B are mutually exclusive events, $P(A \text{ or } B) = P(A) + P(B) = .24 + .28 = .52$.
This probability is not equal to 1.0 because some students will get a grade of C, D, or F.

4.121 Let A = voter is against the discount store and I = voter is indifferent about the discount store. Since these events are mutually exclusive, $P(A \text{ or } I) = P(A) + P(I) = .63 + .17 = .80$.
This probability is not equal to 1.0 because some voters favor letting a major discount store move into their neighborhood.

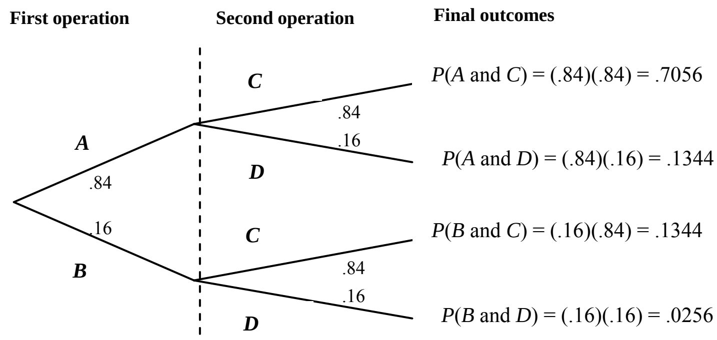
4.122 Let A = first selected corporation makes charitable contributions, B = first selected corporation does not make charitable contributions, C = second selected corporation makes charitable contributions, and D = second selected corporation does not make charitable contributions.

a.



b. P(at most one corporation makes charitable contributions)
$= P(A \text{ and } D) + P(B \text{ and } C) + P(B \text{ and } D) = .2016 + .2016 + .0784 = .4816$

4.123 Let A = first open-heart operation is successful, B = first open-heart operation is not successful, C = second open-heart operation is successful, and D = second open-heart operation is not successful.



P(at least one open-heart operation is successful)

= *P*(*A* and *C*) + *P*(*A* and *D*) + *P*(*B* and *C*) = .7056 + .1344 + .1344 = .9744

Supplementary Exercises

4.124 a. *P*(selected car has a GPS navigation system) = 28/44 = .6364

b. Number of cars which do not have a GPS navigation system = 44 – 28 = 16

P(selected car does not have a GPS navigation system) = 16/44 = .3636

4.125 a. *P*(student selected is a junior) = 9/35 = .2571

b. *P*(student selected is a freshman) = 5/35 = .1429

4.126 Let *S* = psychology major, *C* = communications major, *H* = happy with major, and *U* = unhappy with major.

a. i. $P(H) = 195/250 = .7800$

ii. $P(S) = 100/250 = .4000$

iii. $P(C|H) = 115/195 = .5897$

iv. $P(U|S) = 20/100 = .2000$

v. $P(S \text{ and } H) = P(S)P(H|S) = \left(\frac{100}{250}\right)\left(\frac{80}{100}\right) = .3200$

vi. $P(C \text{ or } U) = P(C) + P(U) - P(C \text{ and } U) = \frac{150}{250} + \frac{55}{250} - \frac{35}{250} = .6800$

b. $P(S) = 100/250 = .4000$ and $P(S|H) = 80/195 = .4103$. Since these two probabilities are not equal, the events "*S*" and "*H*" are dependent. The events "*S*" and "*H*" are not mutually exclusive because they can occur together.

4.127 Let *M* = adult selected is a male, *F* = adult selected is a female, *A* = adult selected prefers watching sports, and *B* = adult selected prefers watching opera.

a. i. $P(B) = 109/250 = .4360$

ii. $P(M) = 120/250 = .4800$

iii. $P(A|F) = 45/130 = .3462$

iv. $P(M|A) = 96/141 = .6809$

v. $P(F \text{ and } B) = P(F)P(B|F) = \left(\frac{130}{250}\right)\left(\frac{85}{130}\right) = .3400$

vi. $P(A \text{ or } M) = P(A) + P(M) - P(A \text{ and } M) = \frac{141}{250} + \frac{120}{250} - \frac{96}{250} = .6600$

b. $P(F) = 130/250 = .5200$ and $P(F|A) = 45/141 = .3191$. Since these two probabilities are not equal, the events "female" and "prefers watching sports" are dependent. The events "female" and "prefers watching sports" are not mutually exclusive because they can occur together.

4.128 Let M = lawyer selected is a male, F = lawyer selected is a female, A = lawyer selected favors capital punishment, and B = lawyer selected opposes capital punishment.

a. i. $P(A) = 45/80 = .5625$

ii. $P(F) = 24/80 = .3000$

iii. $P(B|F) = 11/24 = .4583$

iv. $P(M|A) = 32/45 = .7111$

v. $P(F \text{ and } A) = P(F)P(A|F) = \left(\frac{24}{80}\right)\left(\frac{13}{24}\right) = .1625$

vi. $P(B \text{ or } M) = P(B) + P(M) - P(B \text{ and } M) = \frac{35}{80} + \frac{56}{80} - \frac{24}{80} = .8375$

b. $P(F) = 24/80 = .3000$ and $P(F|B) = 11/35 = .3143$. Since these two probabilities are not equal, the events "female" and "opposes capital punishment" are dependent. The events "female" and "opposes capital punishment" are not mutually exclusive because they can occur together.

4.129 Let A = student selected is an athlete, B = student selected is a nonathlete, F = student selected favors paying college athletes, and N = student selected is against paying college athletes.

a. i. $P(F) = 300/400 = .750$

ii. $P(F|B) = 210/300 = .700$

iii. $P(A \text{ and } F) = P(A)P(F|A) = \left(\frac{100}{400}\right)\left(\frac{90}{100}\right) = .225$

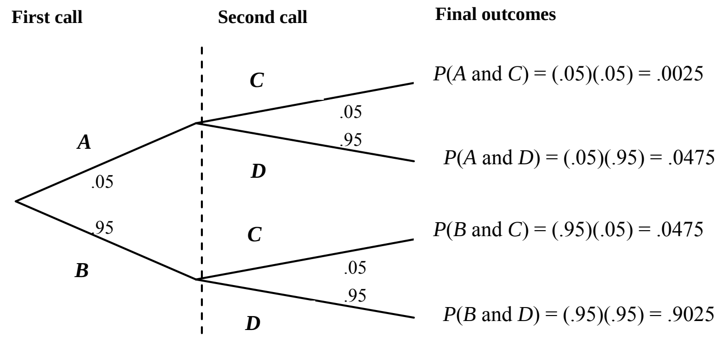
iv. $P(B \text{ or } N) = P(B) + P(N) - P(B \text{ and } N) = \frac{300}{400} + \frac{100}{400} - \frac{90}{400} = .775$

b. $P(A) = 100/400 = .250$ and $P(A|F) = 90/300 = .300$

Since these two probabilities are not equal, the events "athlete" and "should be paid" are dependent. The events "athlete" and "should be paid" are not mutually exclusive because they can occur together.

4.130 Let A = first service call due to customer error, B = first service call due to broken appliance,

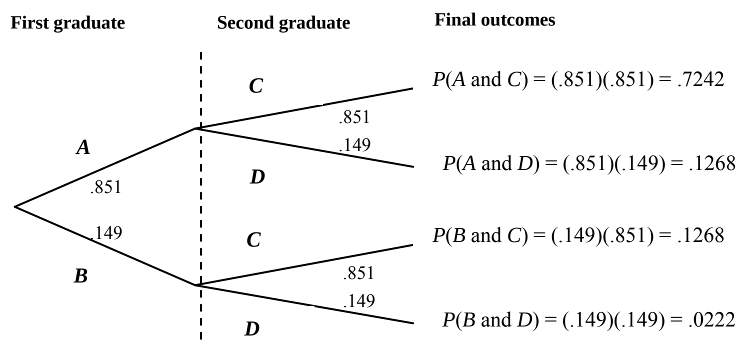
C = second service call due to customer error, and D = second service call due to broken appliance.



a. $P(A \text{ and } C) = .0025$

b. P(at least one service call not due to customer error)
   $= P(A \text{ and } D) + P(B \text{ and } C) + P(B \text{ and } D) = .0475 + .0475 + .9025 = .9975$

4.131 Let A = first graduate selected had job offers before the graduation date, B = first graduate selected did not have job offers before the graduation date, C = second graduate selected did have job offers before the graduation date, and D = second graduate selected did not have job offers before the graduation date.



a. P(both had job offers before the graduation date) $= P(A \text{ and } C) = .7242$

b. P(at most one had job offers before the graduation date)
   $= P(A \text{ and } D) + P(B \text{ and } C) + P(B \text{ and } D) = .1268 + .1268 + .0222 = .2758$

4.132 Let G_1 = first car selected has a GPS system and G_2 = second car selected has a GPS system.

$$P(G_1 \text{ and } G_2) = P(G_1)P(G_2|G_1) = \left(\frac{28}{44}\right)\left(\frac{27}{43}\right) = .3996$$

4.133 Let J_1 = first student selected is a junior and S_2 = second student selected is a sophomore.

$$P(J_1 \text{ and } S_2) = P(J_1)P(S_2|J_1) = \left(\frac{9}{35}\right)\left(\frac{8}{34}\right) = .0605$$

4.134 P(company loses both sources of power)

= *P*(power fails)*P*(generator stops working) = (.30)(.09) = .027

4.135 *P*(both machines are not working properly)
= *P*(first machine is not working properly)*P*(second machine is not working properly)
= (.08)(.06) = .0048

4.136 a. *P*(player wins for first time on tenth bet)

= *P*(player loses first nine bets and wins the tenth) = $\left(\frac{37}{38}\right)^9\left(\frac{1}{38}\right) = .0207$

b. *P*(player loses first 50 bets) = $\left(\frac{37}{38}\right)^{50} = .2636$

c. *P*(player wins at least once in 38 plays)

= 1 – *P*(he loses on all 38 plays) = 1 – $\left(\frac{37}{38}\right)^{38}$ = 1 – .3630 = .6370

4.137 a. There are 26 possibilities for each letter and 10 possibilities for each digit. Hence, there are $26^3 \cdot 10^3 = 17{,}576{,}000$ possible different license places.

b. There are 2 possibilities for the second letter, 26 possibilities for the third letter, and 10 possibilities for each of the two missing numbers. There are 2 × 26 × 10 × 10 = 5200 license plates which fit the description.

4.138 Since the median life of these batteries is 100 hours, the probability that a given battery lasts longer than 100 hours is 1/2. Then,
P(exactly two will last longer than 100 hours) = *P*(first and second last longer but third does not) + *P*(first and third last longer but second does not) + *P*(second and third last longer but first does not)
= (.5)(.5)(.5) + (.5)(.5)(.5) + (.5)(.5)(.5) = .375

4.139 Note: This exercise requires the use of combinations, which students may have studied in algebra. Combinations are covered in Section 5.5.2 in the text.

a. Let *A* = the player's first five numbers match the numbers on the five white balls drawn by the lottery organization, *B* = the player's powerball number matches the powerball number drawn by the lottery organization, and *D* = the player's powerball number does not match the powerball number drawn by the lottery organization. There are ${}_{59}C_5$ ways for the lottery organization to draw five different white balls from a set of 59 balls. Thus the sample space for this phase of the drawing consists of ${}_{59}C_5$ = 5,006,386 equally likely outcomes and $P(A) = 1/({}_{59}C_5)$. Since the sample space for the drawing of the powerball number consists of 39 equally likely outcomes, *P*(*B*) = 1/39 = .0256. Because the powerball number is drawn independently of the five white balls, *A* and *B* are independent events. Therefore,

$$P(\text{player wins jackpot}) = P(A \text{ and } B) = P(A)P(B) = \left(\frac{1}{{}_{59}C_5}\right)\left(\frac{1}{39}\right) = \frac{1}{195{,}249{,}054} = .0000000051$$

b. To win the \$200,000 prize, events A and D must occur. In the sample space of 39 equally likely outcomes for the drawing of the powerball numbers, there are 38 outcomes which do not match the powerball number, and thus result in event D. Hence, $P(D) = 38/39$. Therefore,

P(player wins the \$200,000 prize) = $P(A$ and $D)$

$$= P(A)\,P(D) = \left(\frac{1}{{}_{59}C_5}\right)\left(\frac{38}{39}\right) = \frac{38}{195{,}249{,}054} = .00000019$$

4.140 Let L = left wing engine fails, R = right wing engine fails, and C = central engine fails.

P(crash) = P(C and L, but not R) + P(C and R, but not L) + P(C and R and L)

= (.005)(.008)(.992) + (.005)(.008)(.992) + (.005)(.008)(.008) = .00008

4.141 a. P(sixth marble is red) = 10/20 = .5000

b. P(sixth marble is red) = 5/15 = .3333

c. The probability of obtaining a head on the sixth toss is .5, since each toss is independent of the previous outcomes. Tossing a coin is mathematically equivalent to the situation in part a. Each drawing in part a is independent of previous drawings and the probability of drawing a red marble is .5 each time.

4.142 Let C_1 = first card picked is a club, C_2 = second card picked is a club, D_1 = first card picked is a diamond, and D_2 = second card picked is a diamond.

$$P(\text{you win}) = P(C_1)P(C_2|C_1) + P(D_1)P(D_2|D_1) = \frac{2}{4}\cdot\frac{1}{3}+\frac{2}{4}\cdot\frac{1}{3} = \frac{4}{12} = .3333$$

Do not accept this proposition, since the chance of winning \$10 is only .3333.

4.143 a. The thief has three attempts to guess the correct PIN. Since there are 100 possible numbers in the beginning, the probability that he finds the number on the first attempt is 1/100. Assuming that the first guess is wrong, there are 99 numbers left, etc. Hence,

P(thief succeeds) = 1 – P(thief fails) = 1 – P(thief guessed incorrectly on all three attempts)

$$= 1-\left(\frac{99}{100}\cdot\frac{98}{99}\cdot\frac{97}{98}\right) = 1-\left(\frac{97}{100}\right) = \frac{3}{100} = .030$$

b. Since the first two digits of the four-digit PIN must be 3 and 5, respectively, and the third digit must be 1 or 7, the possible PINs are 3510 to 3519 and 3570 to 3579, a total of 20 possible PINs.

Hence, $P(\text{thief succeeds}) = 1 - P(\text{thief fails}) = 1-\left(\frac{19}{20}\cdot\frac{18}{19}\cdot\frac{17}{18}\right) = 1-\left(\frac{17}{20}\right) = \frac{3}{20} = .150$

4.144 a. P(you must pay gambler) = P(gambler rolls at least one 6 in four tries)

= 1 – P(he rolls no 6 in four tries) = $1 - (5/6)^4 = 1 - .4823 = .5177$

b. Let: E = you obtain at least one double six in 24 rolls,

A_1 = you obtain a double six on the first roll,

A_2 = you obtain a double six on the second roll,

.
.
.

A_{24} = you obtain a double six on the 24th roll

Then, $P(E) = P(A_1 \text{ or } A_2 \text{ or } \cdots \text{ or } A_{24})$. However, $A_1, A_2, \ldots, A_{24}$ are not mutually exclusive, since it is possible to obtain a double 6 on more than one roll. Thus, we cannot find $P(A_1 \text{ or } A_2 \text{ or } \ldots \text{ or } A_{24})$ by simply adding $P(A_1) + P(A_2) + \ldots + P(A_{24})$ as the gambler does to obtain (24)(1/36) = 2/3.

To find $P(E)$ we may use complementary events, as in part a.

The probability of failing to roll a double six on any one attempt is 1 – 1/36 = 35/36.

Hence, $P(E) = 1 - P(\text{you roll no double six in 24 tries}) = 1 - \left(\frac{35}{36}\right)^{24} = 1 - .5086 = .4914$

Since your chance of winning is less than 50%, the gambler has the advantage and you should not accept his proposition.

4.145 Let J_1 = your friend selects the first jar, J_2 = your friend selects the second jar, R = your friend selects a red marble, and G = your friend selects a green marble.

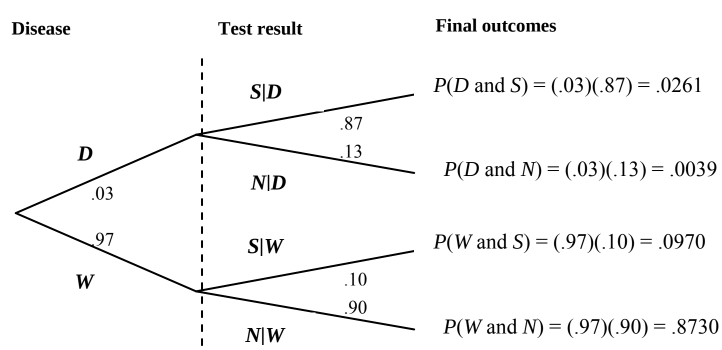
a. $P(R) = P(J_1)P(R|J_1) + P(J_2)P(R|J_2) = (1/2)(5/10) + (1/2)(5/10) = .5000$

b. $P(R) = P(J_1)P(R|J_1) + P(J_2)P(R|J_2) = (1/2)(2/4) + (1/2)(8/16) = .5000$

c. Put one red marble in one jar, and the rest of the marbles in the other jar. Then,

$P(R) = P(J_1)P(R|J_1) + P(J_2)P(R|J_2) = (1/2)(1) + (1/2)(9/19) = .7368$

4.146 Let S = the test result is positive, N = the test result is negative, D = the patient selected has the disease, and W = the patient selected does not have the disease.



a. $P(D \text{ and } S) = .0261$

b. $P(W \text{ and } S) = .0970$

c. $P(S) = P(D \text{ and } S) + P(W \text{ and } S) = .0261 + .0970 = .1231$

d. $P(D|S) = P(D \text{ and } S) / P(S) = .0261/.1231 = .2120$

4.147 a. Let E = neither topping is anchovies, A = customer's first selection is not anchovies, and B = customer's second selection is not anchovies,.

For A to occur, the customer may choose any of 11 toppings from the 12 available.

Thus $P(A) = \frac{11}{12}$. For B to occur, given that A has occurred, the customer may choose any of 10 toppings from the remaining 11 and $P(B|A) = \frac{10}{11}$. Therefore, $P(E) = P(A \text{ and } B) =$

$P(A)P(B|A) = \left(\frac{11}{12}\right)\left(\frac{10}{11}\right) = .8333$

b. Let C = pepperoni is one of the toppings. Then $\overline{C}$ = neither topping is pepperoni. By a similar argument used to find $P(E)$ in part a, we obtain $P(\overline{C}) = \left(\frac{11}{12}\right)\left(\frac{10}{11}\right) = .8333$. Then,

$P(C) = 1 - P(\overline{C}) = 1 - .8333 = .1667$.

4.148 Let C = auto policy holders with collision coverage and U = auto policy holders with uninsured motorist coverage.

a. $P(C \text{ or } U) = P(C) + P(U) - P(C \text{ and } U)$.

Hence, $P(C \text{ and } U) = P(C) + P(U) - P(C \text{ or } U) = .80 + .60 - .93 = .47$

Thus, 47% of the policy holders have both collision and uninsured motorist coverage.

b. The event "$\overline{C}$ and $\overline{U}$" is the complement of the event "C or U".

Hence, $P(\overline{C} \text{ and } \overline{U}) = 1 - P(C \text{ or } U) = 1 - .93 = .07$

Thus, 7% of the policy holders have neither collision nor uninsured motorist coverage.

c. The group of policy holders who have collision but not uninsured motorist coverage may be formed by considering the policy holders that have collision, then removing those who have both collision and uninsured motorist coverage. Hence,

$P(C \text{ and } \overline{U}) = P(C) - P(C \text{ and } U) = .80 - .47 = .33$

Thus, 33% of policy holders have collision but not uninsured motorist coverage.

4.149 a. $P(\text{win}) = \frac{1}{10\times10\times10\times10} = .0001$

b. i. Since all four digits are unique and order does not matter, $P(\text{win}) = \frac{1}{{}_{10}C_4} = .0048$.

ii. There are ${}_{10}C_3$ ways to choose 3 distinct numbers, and for each of those sets of 3 numbers there are 3 digits that can be repeated. Then, $P(\text{win}) = \dfrac{1}{{}_{10}C_3 \times 3} = .0028$.

iii. There are ${}_{10}C_2$ ways to choose 2 distinct numbers and both of the numbers must be repeated. Then, $P(\text{win}) = \dfrac{1}{{}_{10}C_2} = .0222$.

iv. There are ${}_{10}C_2$ ways to choose 2 distinct numbers, and for each of those sets of 2 numbers there are 2 digits that can be repeated. Then, $P(\text{win}) = \dfrac{1}{{}_{10}C_2 \times 2} = .0111$.

4.150 Let D_1 = the first oven is defective, D_2 = the second oven is defective, D_3 = the third oven is defective, D_4 = the fourth oven is defective, D_5 = the fifth oven is defective, N_1 = the first oven is not defective, N_2 = the second oven is not defective, N_3 = the third oven is not defective, N_4 = the fourth oven is not defective, and N_5 = the fifth oven is not defective. The ovens will be purchased if one of the following outcomes is achieved: $N_1N_2N_3N_4N_5$, $D_1N_2N_3N_4N_5$, $N_1D_2N_3N_4N_5$, $N_1N_2D_3N_4N_5$, $N_1N_2N_3D_4N_5$, $N_1N_2N_3N_4D_5$. There are 8 defective and 92 non-defective ovens in the batch. Then,
P(ovens are purchased) = $P(N_1N_2N_3N_4N_5) + P(D_1N_2N_3N_4N_5) + P(N_1D_2N_3N_4N_5) + P(N_1N_2D_3N_4N_5) + P(N_1N_2N_3D_4N_5) + P(N_1N_2N_3N_4D_5)$ = (92/100)(91/99)(90/98)(89/97)(88/96) + (8/100)(92/99)(91/98)(90/97)(89/96) + (92/100)(8/99)(91/98)(90/97)(89/96) + (92/100)(91/99)(8/98)(90/97)(89/96) + (92/100)(91/99)(90/98)(8/97)(89/96) + (92/100)(91/99)(90/98)(89/97)(8/96) = .9501

4.151 Let W_1 = the first machine on the first line works, W_2 = the second machine on the first line works, W_3 = the first machine on the second line works, W_4 = the second machine on the second line works, D_1 = the first machine on the first line does not work, D_2 = the second machine on the first line does not work, D_3 = the first machine on the second line does not work, and D_4 = the second machine on the second line does not work.

a. $P(W_1W_2W_3W_4) = P(W_1)P(W_2)P(W_3)P(W_4) = (.98)(.96)(.98)(.96) = .8851$

b. P(at least one machine in each production line is not working properly)
$= P(W_1D_2W_3D_4) + P(W_1D_2D_3W_4) + P(D_1W_2W_3D_4) + P(D_1W_2D_3W_4) + P(W_1D_2D_3D_4) + P(D_1W_2D_3D_4) + P(D_1D_2W_3D_4) + P(D_1D_2D_3W_4) + P(D_1D_2D_3D_4)$ = (.98)(.04)(.98)(.04) + (.98)(.04)(.02)(.96) + (.02)(.96)(.98)(.04) + (.02)(.96)(.02)(.96) + (.98)(.04)(.02)(.04) + (.02)(.96)(.02)(.04) + (.02)(.04)(.98)(.04) + (.02)(.04)(.02)(.96) + (.02)(.04)(.02)(.04) = .0035

Self-Review Test

1. a **2.** b **3.** c **4.** a **5.** a **6.** b

7. c **8.** b **9.** b **10.** c **11.** b

12. Total outcomes = 4 × 3 × 5 × 2 = 120

13. a. P(job offer selected is from the insurance company) = 1/3 = .3333

b. P(job offer selected is not from the accounting firm) = 2/3 = .6667

14. a. P(out of state) = 125/200 = .6250 and P(out of state | female) = 70/110 = .6364. Since these two probabilities are not equal, the two events are dependent. Events "female" and "out of state" are not mutually exclusive because they can occur together.

b. i. In 200 students, there are 90 males. Hence, P(a male is selected) = 90/200 = .4500.

ii. There are a total of 110 female students and 70 of them are out of state students. Hence, P(out of state | female) = 70/110 = .6364.

15. P(out of state or female) = P(out of state) + P(female) – P(out of state and female)

= P(out of state) + P(female) – P(out of state) P(female |out of state)= $\frac{125}{200}+\frac{110}{200}-\left(\frac{110}{200}\right)\left(\frac{70}{110}\right)=.825$

16. Let S_1 = first student selected is from out of state and S_2 = second student selected is from out of state.

Then, $P(S_1 \text{ and } S_2) = P(S_1)P(S_2|S_1) = \left(\frac{125}{200}\right)\left(\frac{124}{199}\right) = .3894$

17. Let F_1 = first adult selected has experienced a migraine headache, N_1 = first adult selected has never experienced a migraine headache, F_2 = second adult selected has experienced a migraine headache, and N_2 = second adult selected has never experienced a migraine headache. Note that the two adults are independent. From the given information: $P(F_1) = .35$ and $P(F_2) = .35$. Hence, $P(N_1) = 1 - .35 = .65$ and $P(N_2) = 1 - .35 = .65$. Then, $P(N_1 \text{ and } N_2) = P(N_1)P(N_2) = (.65)(.65) = .4225$.

18. $P(A) = 8/20 = .400$. The complementary event of A is that the selected marble is not red, that is, the selected marble is either green or blue. Hence, $P(\bar{A}) = 1 – .400 = .600$.

19. Let M = male, F = female, W = works at least 10 hours, and N = does not work 10 hours.

a. $P(M \text{ and } W) = P(M)P(W) = (.45)(.62) = .279$

b. $P(F \text{ or } W) = P(F) + P(W) – P(F \text{ and } W) = .55 + .62 – (.55)(.62) = .829$

20. a. i. $P(Y) = (77 + 104)/506 = .3577$

ii. $P(Y|W) = 104/(104 + 119 + 34) = .4047$

iii. $P(W \text{ and } N) = P(W)P(N/W) = \left(\frac{257}{506}\right)\left(\frac{119}{257}\right) = .2352$

iv. $P(N \text{ or } M) = P(N) + P(M) – P(N \text{ and } M) = \frac{66}{506}+\frac{249}{506}-\frac{32}{506}=.5593$

b. $P(W) = 257/506 = .5079$ and $P(W|Y) = 104 / 181 = .5746$. Since these two probabilities are not equal, the events "woman" and "yes" are dependent. The events "woman" and "yes" are not mutually exclusive because they can occur together.